The impact of present Δm_s measurement on UT fit

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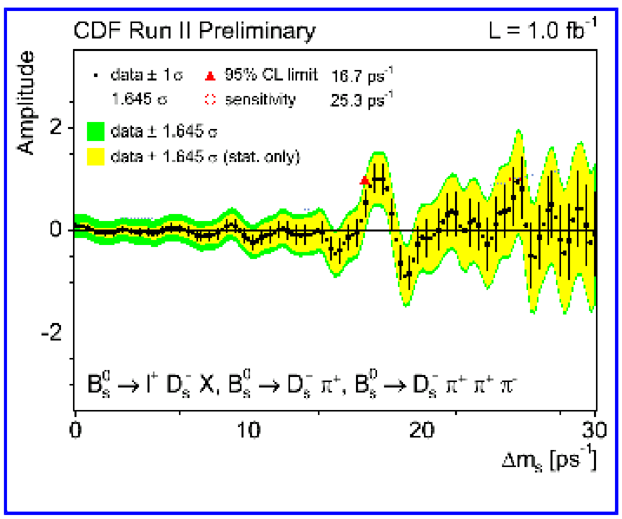
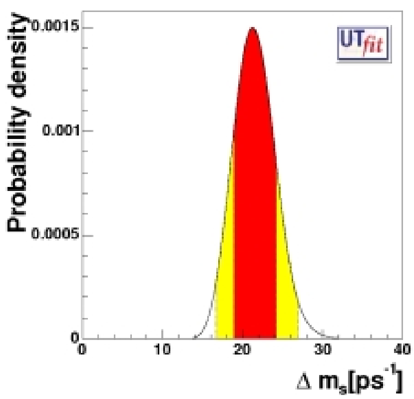
Experimental Status (Winter 2006)

The CDF collaboration reported the first measurement of Δm_s, which provides a test of the Standard Model through the comparison to the indirect determination, obtained from a combined fit to all the remaining constraints (**$|V_{ub}/V_{cb}|$, Δm_d, ε_K, sin2β, cos2β, α and γ**).



Amplitude as a function of Δm_s from CDF

This comparison is a crucial test for the Standard Model and it plays in the b → s sector the same role of sin2β measurements in b → d sector few years ago. At this point, the two informations do not show any significant evidence of disagreement



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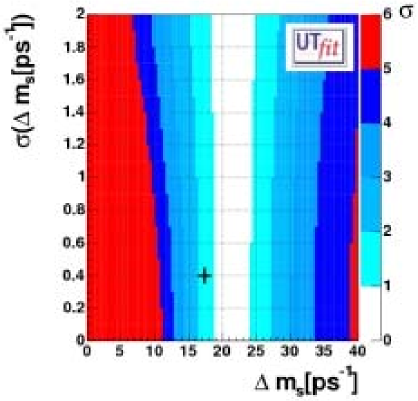

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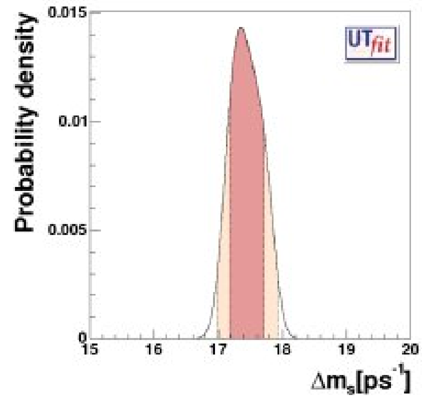

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INDIRECT MEASUREMENT:

$\Delta m_s = (21.5 \pm 2.6)\ ps^{-1}$

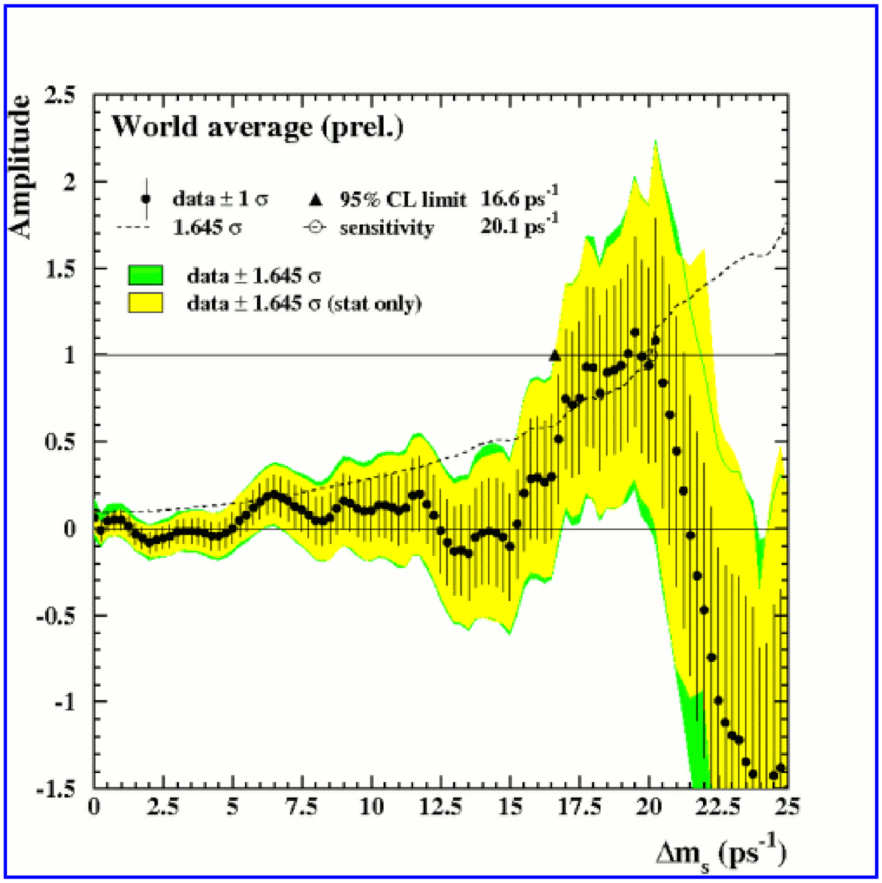
Pull distribution
DIRECT MEASUREMENT:

$\Delta m_s = (17.35 \pm 0.25)\ ps^{-1}$

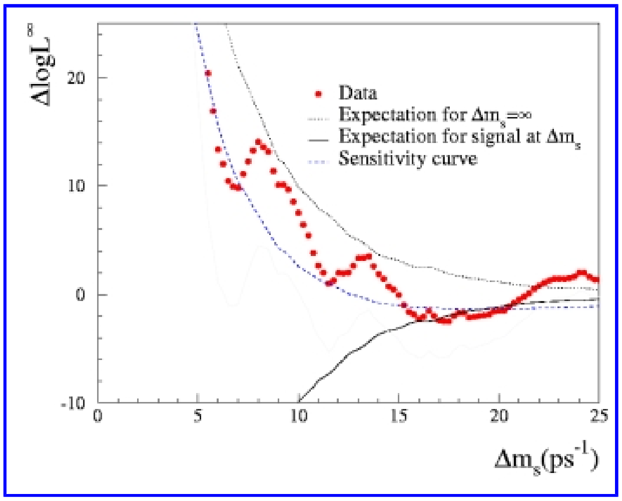
The amplitude method

The amplitude method has been introduced by H.G. Moser and A.Roussarie and is described in Nucl. Instrum. Meth. A384 (1997) 491. with the aim of setting limits on Δm_s and to combine results from different analyses.

The method consists in modifying the equation describing the probability that a B^0 meson oscillates into a $\overline{B}^0$ in the following way : **$1 \pm \cos \Delta m_s t \Rightarrow 1 \pm A \cos \Delta m_s t$.** A and σ_A are measured at fixed values of Δm_s instead of Δm_s itself. In case of a clear oscillation signal, at a given frequency, the amplitude should be compatible with A = 1 at this frequency. With this method it is easy to set a limit. The values of Δm_s excluded at 95% C.L. are those satisfying the condition $A(\Delta m_s) + 1.645\ \sigma_A\ (\Delta m_s) < 1$.
Furthermore the sensitivity of the experiment can be defined as the value of Δm_s corresponding to $1.645\ \sigma_A\ (\Delta m_s) = 1$ (taking $A(\Delta m_s) = 0$), namely supposing that the ``true'' value of Δm_s is well above the measurable value.
For example we show, the combined result of LEP/SLD/CDF/D0 analyses (situation at Moriond06) (see HFAG) is shown in Figure (plot on the left) below



Amplitude as a function of Δm_S (HFAG WA, not including new CDF)



$\Delta \log L^{infinity}(\Delta m_S)$.

The Likelihood ratio method

The 95% C.L. limit and the sensitivity, are useful to summarize the results of the analysis. However to include Δm_S in a CKM fit and to determine probability regions for the Unitarity Triangle parameters, continuous information about the degree of

exclusion of a given value of Δm_s is needed.
The log-likelihood values (it is the log-likelihood referenced to its value obtained for Δm_s=infinity) can be easily deduced from A and σ_A using the expressions :

$$\Delta \log \mathcal{L}^{\infty}(\Delta M_s) = \frac{1}{2}\left[\left(\frac{A-1}{\sigma_A}\right)^2 - \left(\frac{A}{\sigma_A}\right)^2\right] = \left(\frac{1}{2} - A\right)\frac{1}{\sigma_A^2}$$

$$\Delta \log \mathcal{L}^{\infty}(\Delta M_s)_{\rm mix} = -\frac{1}{2}\frac{1}{\sigma_A^2}$$

$$\Delta \log \mathcal{L}^{\infty}(\Delta M_s)_{\rm nomix} = \frac{1}{2}\frac{1}{\sigma_A^2}$$

The last two equations give the average log-likelihood value for Δm_s corresponding to the true oscillation frequency (mixing case) and for Δm_s being far from the oscillation frequency ($|\Delta m_s - \Delta m_s^{true}| >> \Gamma/2$, no-mixing case). Γ is here the full width at half maximum of the amplitude distribution in case of a signal; typically $\Gamma \sim 1/\tau(B_s)$. The $\Delta \log L^{infinity}(\Delta m_s)$ plot from the world average is shown in the above Figure (plot on the right)
The Likelihood Ratio R is defined as :

$$\mathrm{R}(\Delta M_s) = e^{-\Delta \log \mathcal{L}^{\infty}(\Delta M_s)} = \frac{\mathcal{L}(\Delta M_s)}{\mathcal{L}(\Delta M_s = \infty)}$$

It has been shown (Yellow Book CERN-EP/2003-002 hep-ph/0304132 pages 182-190) that in the Bayesian approach the correct method to include this information is the Likelihood ratio method. A similar method to incorporate results from mixing can be found in hep-ph/9607469.
