Theory of $\bar{B} \rightarrow X_u l^- \bar{\nu}$ decays and $|V_{ub}|$

Björn O. Lange

Center for Theoretical Physics

Massachusetts Institute of Technology
Outline

1. Direct calculation of partial decay rates in the “shape-function region”.

2. Relations between $\bar{B} \rightarrow X_u l^- \bar{\nu}$ partial rates and $\bar{B} \rightarrow X_s \gamma$ photon spectrum.

Motivation

Extraction of $|V_{ub}|$, UT side opposite $\beta$. 
Inclusive semileptonic $B$ decays

Kinematics of $\bar{B} \rightarrow X_u l^- \bar{\nu}$

Three independent variables, e.g. energy $E_l$ of the charged lepton, and light-cone momenta $P_{\pm} = E_X \mp |\vec{P}_X|$ of the final hadronic state.

![Graph showing kinematic constraints]

- Large charm background: $P_+ P_- > M_D^2$
- No charm background: $P_+ P_- < M_D^2$
Inclusive semileptonic $B$ decays

- Accurate measurements only for charmfree region.
- Typically where $P_+ \sim O(\Lambda_{QCD})$, and $P_- \sim O(m_b)$.
- Separation of physics at different scales:
  \[ \mu_h \sim m_b, \quad \mu_i \sim \sqrt{m_b \Lambda_{QCD}}, \quad \Lambda_{QCD} \]
- Based on universal **QCD-factorization** formula

\[ d\Gamma = HJ \otimes \hat{S} + \frac{1}{m_b} H_i' J_i' \otimes \hat{S}_i' + \ldots \]

Inclusive semileptonic $B$ decays

The fully differential decay rate can be written as

$$\frac{d^3\Gamma_u}{dP_+ dP_- dP_l} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} U_y(\mu_h, \mu_i) (M_B - P_+) \left[ (P_- - P_l)(M_B - P_- + P_l - P_+) \mathcal{F}_1 ight. + (M_B - P_-)(P_- - P_+) \mathcal{F}_2 + \left. (P_- - P_l)(P_l - P_+) \mathcal{F}_3 \right]$$

without explicit reference to partonic quantities. Here, $y = (P_- - P_+)/(M_B - P_+)$.

$\mathcal{F}_i$ are factorized in a QCD $\rightarrow$ SCET $\rightarrow$ HQET matching procedure:

- **Leading Power:** [Bauer, Manohar; Phys. Rev. D 70, 034024 (2004);

  $$\mathcal{F}^{(0)}_i(P_+, y) = H_{ui}(y, \mu_h) \int_0^{P_+} d\hat{\omega} \, y m_b \, J(y m_b (P_+ - \hat{\omega}), \mu_i) \hat{S}(\hat{\omega}, \mu_i)$$

- **Subleading Power corrections from subleading shape functions**
  (hadronic corrections) and kinematical corrections.

Björn O. Lange (MIT)
Why do this?

- Systematic separation of effects at different scales.
- EFTs have more symmetry $\Rightarrow \hat{S}(\hat{\omega})$ unique!
  $\Rightarrow$ For example, $\bar{B} \rightarrow X_s \gamma$ spectrum factorizes similarly. [Neubert, Eur. Phys. J. C 40, 165 (2005)]
- Resummation of large logs.

$$\alpha_s^n \ln^k \frac{\mu_h}{\mu_i}, \quad k = 2n, 2n - 1, \ldots \quad \text{("Sudakov factor")}$$

1-loop matching, 2-loop running, 3-loop cusp anomalous dimension
Experimentalists like to have predictions for entire phase-space, so we need . . .

**Connection with “OPE region”**

\[
\int_0^\Delta d\hat{\omega} \ f(\hat{\omega}) \hat{S}(\hat{\omega}) , \quad \Delta \gg \Lambda_{QCD}
\]

- can perform OPE in \( \Lambda_{QCD}/\bar{\Delta} , \alpha_s(\bar{\Delta}) \), where \( \bar{\Delta} = \Delta - \bar{\Lambda} \).


- sensitive to moments of \( \hat{S} \).

**But:** different power counting, nontrivial transition.
Inclusive semileptonic $B$ decays

**Anatomy of partial rate** $\int_0^\Delta dP_+ \frac{d\Gamma}{dP_+}$

$\Delta \sim \Lambda_{\text{QCD}}$ (SF region), $\Delta \gg \Lambda_{\text{QCD}}$ (OPE region)

When integrating the tree-level, leading-power factorized expression over a large phase space $\rightarrow$ tree-level, leading-power OPE result. Also feeds into subleading power on the OPE side. (Via moments of the leading shape function.)
Inclusive semileptonic $B$ decays

Anatomy of partial rate $\int_{0}^{\Delta} dP_{+} \frac{d\Gamma}{dP_{+}}$

This pattern repeats itself at \( \Delta \sim \Lambda_{QCD} \) (SF region), \( \Delta \gg \Lambda_{QCD} \) (OPE region) at each higher order in power counting.

(E.g. first subleading shape functions have zero norm, but nonzero first moment.)

[Bosch, Neubert and Paz; JHEP 0411, 073 (2004)]
[Beneke, Campanario, Mannel, Pecjak; JHEP 0506, 071 (2005)]
Inclusivesemileptonic $B$ decays

**Anatomy of partial rate** \[ \int_0^\Delta dP \frac{d\Gamma}{dP_+} \]

\[ \Delta \sim \Lambda_{QCD} \text{ (SF region)} \] , \[ \Delta \gg \Lambda_{QCD} \text{ (OPE region)} \]

Including **radiative corrections** to the factorized leading-power decay rate and increasing $\Delta$ does not reproduce the OPE result. Starting at $\mathcal{O}(\alpha_s)$ some terms are missing because they are suppressed for small $\Delta$. These are “kinematical corrections”.

[De Fazio, Neubert; JHEP 9906, 017 (1999)]

---

**Björn O. Lange (MIT)**
By including all contributions in the purple box the difference between the factorized expressions (applied for large $\Delta$) and a standard OPE is analytically of order $(1/m_b)^3$ and numerically negligible. This has been incorporated in

Inclusive semileptonic $B$ decays

**Modelling the shape functions**

Predictions require functional forms for the shape functions.

- **Leading shape function** extracted from $\bar{B} \rightarrow X_s \gamma$ photon spectrum.

  For this absorb subleading shape functions into $\hat{S}$.

Can $\bar{B} \rightarrow X_c l^- \bar{\nu}$ spectra help? [Boos, Feldmann, Mannel, Pecjak; hep-ph/0512157]

(In particular $U = P_+ - m_c^2/p_-$ spectrum?)

Only available information for subleading shape functions are their first few moments. So we model and estimate uncertainty.
Inclusive semileptonic $B$ decays

Example: $P_+$ spectrum

The $P_+$ spectrum extended to large values of $P_+$. The thin solid line denotes the leading-power prediction, the dashed line depicts first-order power corrections, the dash-dotted line shows second-order power corrections, and the thick solid line is their sum.
Example: Cut on $P_+ \leq 0.65 \text{ GeV}$

- The central value assumes perfect extraction of $\hat{S}(\hat{\omega})$.

$$
\Gamma_u(0.65 \text{ GeV}) = (45.3 \pm 2.5_{\text{pert}} \pm 1.5_{\text{hadr}} \pm 1.3_{\text{WA}})|V_{ub}|^2 \text{ ps}^{-1}
$$
**Example: Cut on** \( P_+ \leq 0.65 \text{ GeV} \)

- The central value assumes perfect extraction of \( \hat{S}(\hat{\omega}) \).
- Perturbative uncertainty from NLO approximation. Will improve with the new 2-loop result for the jet function [Becher, Neubert, hep-ph/0603140].

\[
\Gamma_u(0.65 \text{ GeV}) = (45.3 \pm 2.5\text{[pert]} \pm 1.5\text{[hadr]} \pm 1.3\text{[WA]})|V_{ub}|^2 \text{ps}^{-1}
\]
Inclusive semileptonic $B$ decays

**Example: Cut on** $P_+ \leq 0.65$ GeV

- The central value assumes perfect extraction of $\hat{S}(\hat{\omega})$.
- Perturbative uncertainty from NLO approximation. Will improve with the new 2-loop result for the jet function [Becher, Neubert, hep-ph/0603140].
- Hadronic uncertainty from subleading shape functions: scan over many models.
  (Combinatorically 700+ models)

\[
\Gamma_u(0.65 \text{ GeV}) = (45.3 \pm 2.5 \text{[pert]} \pm 1.5 \text{[hadr]} \pm 1.3 \text{[WA]})|V_{ub}|^2 \text{ ps}^{-1}
\]

Björn O. Lange (MIT)
Example: Cut on $P_+ \leq 0.65$ GeV

- The central value assumes perfect extraction of $\hat{S}(\hat{\omega})$.
- Perturbative uncertainty from NLO approximation. Will improve with the new 2-loop result for the jet function [Becher, Neubert, hep-ph/0603140].
- Hadronic uncertainty from subleading shape functions: scan over many models.
  (Combinatorically 700+ models)
- Weak annihilation estimated $\sim 1.8\%$ of total rate. [Tom Meyer, analysis of CLEO data.]

\[
\Gamma_u(0.65 \text{ GeV}) = (45.3 \pm 2.5 \text{ [pert]} \pm 1.5 \text{ [hadr]} \pm 1.3 \text{ [WA]})|V_{ub}|^2 \text{ ps}^{-1}
\]
Inclusive semileptonic $B$ decays

**Example: Cut on $P_+ \leq 0.65$ GeV**

- The central value assumes perfect extraction of $\hat{S}(\hat{\omega})$.
- Perturbative uncertainty from NLO approximation. Will improve with the new 2-loop result for the jet function [Becher, Neubert, hep-ph/0603140].
- Hadronic uncertainty from subleading shape functions: scan over many models. (Combinatorically 700+ models)
- Weak annihilation estimated $\sim 1.8\%$ of total rate. [Tom Meyer, analysis of CLEO data.]
  Can be tested with cut on high $q^2$. [B.O.L., Neubert, Paz: Phys. Rev. D 72, 073006 (2005)]

$$\Gamma_u(0.65 \text{ GeV}) = (45.3 \pm 2.5 \text{ [pert]} \pm 1.5 \text{ [hadr]} \pm 1.3 \text{ [WA]}) |V_{ub}|^2 \text{ ps}^{-1}$$
Inclusive semileptonic $B$ decays

**Briefly: a different approach**

Recently a calculation called “Dressed Gluon Exponentiation” appeared.

[Andersen, Gardi; JHEP 0506 030 (2005); JHEP 0601 097 (2006)]

- Instead of $B$-meson state, DGE uses on-shell $b$-quark “dressed” with gluons.
- Nevertheless, kinematic range extends beyond partonic phase space.
- Assumes models for exact anomalous dimensions of jet and soft functions, motivated by large $\beta_0$ limit and cancellation of certain renormalon ambiguities.
- Claims that then there is no need for non-perturbative functions until very endpoint of spectrum.

**This is a model calculation.**

Björn O. Lange (MIT)

Vancouver, April 9-12, 2006

Flavor Physics & CP Violation
Shape-function free relations
Motivation

Extracting the leading shape function from $\bar{B} \to X_s \gamma$ data is far from trivial.
Motivation

Extracting the leading shape function from $\bar{B} \rightarrow X_s \gamma$ data is far from trivial.

This motivates to eliminate the “middle man” and use the spectrum directly.
Motivation

Extracting the leading shape function from $\bar{B} \to X_s \gamma$ data is far from trivial.

This motivates to eliminate the “middle man” and use the spectrum directly.

Avoid worries from resonances, SF parameterization, etc.
Motivation

Extracting the leading shape function from $\bar{B} \rightarrow X_s \gamma$ data is far from trivial.

This motivates to eliminate the “middle man” and use the spectrum directly.

Avoid worries from resonances, SF parameterization, etc.

Complementary way to the direct calculation of partial decay rates.
Motivation

Extracting the leading shape function from $\bar{B} \rightarrow X_s \gamma$ data is far from trivial.

This motivates to eliminate the “middle man” and use the spectrum directly.

Avoid worries from resonances, SF parameterization, etc.

Complementary way to the direct calculation of partial decay rates.

We can make full use of QCD factorization theorems for the spectra!
Inclusive semileptonic $B$ decays

Elimination of the shape function.

$B \rightarrow X_u l^- \nu$ 

$B \rightarrow X_s \gamma$

\[
P_+ = M_B - 2E_\gamma
\]

\[\Gamma_u \bigg|_{cut} = |V_{ub}|^2 \int_0^\Delta dP_+ \, W(\Delta, P_+) \frac{1}{\Gamma_s} \frac{d\Gamma_s}{dP_+} + \text{pow. corr.}\]

Björn O. Lange (MIT)
• The idea was put forward in [Neubert, Phys. Rev. D 49, 4623 (1994)],

• further work done for cuts on lepton energy


  [B.O.L., Neubert, Paz, JHEP 0510, 084 (2005)],

• any [B.O.L., JHEP 0601, 104 (2006)].
Inclusive semileptonic $B$ decays

\[ \Gamma_u \bigg|_{\text{cut}} = |V_{ub}|^2 \int_0^\Delta dP_+ \ W(\Delta, P_+) \ \frac{1}{\Gamma_s} \frac{d\Gamma_s}{dP_+} + \text{pow. corr.} \]

- Using QCD factorization theorems
  \[ d\Gamma^{(0)} \sim H(\mu_h) J(\mu_i) \otimes \hat{S}(\mu_i) \]
  we can show that
  \[ W^{(0)} \sim H(\mu_h) \otimes Y(\mu_i), \]
  where the kernel $Y$ is derived from the jet function $J$.

  [To complete 2-loop order!]

- The cut is encoded in the convolution between $H$ and $Y$.

  [Automation!]
Inclusive semileptonic $B$ decays

For Reference:

\[ \bar{B} \rightarrow X_u l^- \bar{\nu} \]

\[ \bar{B} \rightarrow X_s \gamma \]

\[ P_+ = M_B - 2E_\gamma \]
Figure 2: Examples of the weight function for different kinematic cuts. LEFT: Cutting on $P_+ \leq \Delta = 0.66$ GeV and $E_l > E_0$. From top to bottom the four functions are for $E_0 = 0$, $E_0 = 1$ GeV, $E_0 = 2$ GeV, and $E_0 = (M_B - \Delta)/2$. RIGHT: Cutting on $M_X \leq M_0 = 1.7$ GeV, $q^2 > q_0^2$, and $E_l > 1$ GeV. The three functions are for $q_0^2 = 0$ (top), $q_0^2 = 8$ GeV$^2$ (middle), and $q_0^2 = (M_B - M_0)^2$ (bottom).
For cuts on hadronic mass need photon spectrum over wide range

Fortunately, the weight function is small there.

Cutting away further events in the low-\( P_+ \) region makes no sense here.

“Pure” cut on \( P_+ \) most promising.

Cutting soft leptons away doesn’t hurt much.

We are looking forward to the first implementation by BaBar and Belle.
Example: Cut on $P_+ \leq 0.65$ GeV

- Perturbative contribution and uncertainty from NNLO @ $\mu_i$ as important as NLO @ $\mu_h$.

$$\Gamma_u(0.65 \text{ GeV}) = (46.5 \pm 1.4 \text{ [pert]} \pm 1.8 \text{ [had]} \pm 1.8 \text{ [mb]} \pm 0.8 \text{ [pars]} \pm 2.8 \text{ [norm]} ) |V_{ub}|^2 \text{ ps}^{-1}$$

$$= (46.5 \pm 4.1) |V_{ub}|^2 \text{ ps}^{-1}, \quad (45)$$
Example: Cut on $P_+ \leq 0.65$ GeV

- Perturbative contribution and uncertainty from NNLO @ $\mu_i$ as important as NLO @ $\mu_h$.
- Hadronic uncertainty from subleading shape functions: scan over many models.

\[
\Gamma_u(0.65 \text{ GeV}) = (46.5 \pm 1.4 \text{ [pert]} \pm 1.8 \text{ [had]} \pm 1.8 \text{ [mb]} \pm 0.8 \text{ [pars]} \pm 2.8 \text{ [norm]}) |V_{ub}|^2 \text{ ps}^{-1}
\]

\[
= (46.5 \pm 4.1) |V_{ub}|^2 \text{ ps}^{-1} , \tag{45}
\]
Example: Cut on $P_+ \leq 0.65$ GeV

- Perturbative contribution and uncertainty from NNLO @ $\mu_i$ as important as NLO @ $\mu_h$.
- Hadronic uncertainty from subleading shape functions: scan over many models.
- Input parameter variation: can be improved with better $m_b$ determination.

\[
\Gamma_u(0.65 \text{ GeV}) = (46.5 \pm 1.4 \text{ [pert]} \pm 1.8 \text{ [hadr]} \pm 1.8 \text{ [mb]} \pm 0.8 \text{ [pars]} \pm 2.8 \text{ [norm]}) |V_{ub}|^2 \text{ ps}^{-1} \\
= (46.5 \pm 4.1) |V_{ub}|^2 \text{ ps}^{-1}, \tag{45}
\]
Example: Cut on $P_+ \leq 0.65 \text{ GeV}$

- Perturbative contribution and uncertainty from NNLO @ $\mu_i$ as important as NLO @ $\mu_h$.
- Hadronic uncertainty from subleading shape functions: scan over many models.
- Input parameter variation: can be improved with better $m_b$ determination.
- Uncertainty from norm of spectrum: can be improved with higher-order calculation.


\[
\Gamma_u(0.65 \text{ GeV}) = (46.5 \pm 1.4 \text{ [pert]} \pm 1.8 \text{ [hadr]} \pm 1.8 \text{ [mb]} \pm 0.8 \text{ [pars]} \pm 2.8 \text{ [norm]}) |V_{ub}|^2 \text{ ps}^{-1} \\
= (46.5 \pm 4.1) |V_{ub}|^2 \text{ ps}^{-1},
\]

(45)
Example: Cut on $P_+ \leq 0.65 \text{ GeV}$

- Perturbative contribution and uncertainty from NNLO @ $\mu_i$ as important as NLO @ $\mu_h$.
- Hadronic uncertainty from subleading shape functions: scan over many models.
- Input parameter variation: can be improved with better $m_b$ determination.
- Uncertainty from norm of spectrum: can be improved with higher-order calculation.


\[
\Gamma_u(0.65 \text{ GeV}) = (46.5 \pm 1.4 \text{ [pert]} \pm 1.8 \text{ [hadr]} \pm 1.8 \text{ [mb]} \pm 0.8 \text{ [pars]} \pm 2.8 \text{ [norm]}) |V_{ub}|^2 \text{ ps}^{-1}
= (46.5 \pm 4.1) |V_{ub}|^2 \text{ ps}^{-1},
\]

Overall theoretical error on $|V_{ub}|$ thus far: 4-5%.
Inclusive semileptonic $B$ decays

Summary:

$$d\Gamma = H J \otimes \hat{S} + \frac{1}{m_b} H' J'_i \otimes \hat{S}'_i + \ldots$$

$$W = H_G H_u \otimes Y + \ldots$$

End of Summary

What could be next? [Just some thoughts, no promises implied!]

- Perturbative corrections:
  - $J$ at 2-loop (most important). $Y$ already at 2-loop.
  - $H$ at 2-loop: matching $V - A$ current to SCET current. For $\bar{B} \to X_s \gamma$ at NNLO major effort.
  - Resummation (Sudakov) improvement to next level requires 3- and 4-loop running.
  - Perturbative corrections at subleading power.

- Shape functions:
  - More terms in OPE of shape-function moments? (Mainly for MSOPE)
  - More terms in OPE of subleading shape-function moments?

Thank you.