Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time

11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c$^2$

Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/c$^2$
<table>
<thead>
<tr>
<th>Measurement</th>
<th>Fit</th>
<th>$\Delta O^\text{meas} - O^\text{fit} / \sigma^\text{meas}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha^{(5)}_{\text{had}}(m_Z)$</td>
<td>$0.02761 \pm 0.00036$</td>
<td>0.02770</td>
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<tr>
<td>$m_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
<td>91.1874</td>
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<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
<td>2.4965</td>
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<tr>
<td>$\sigma_{\text{had}}$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
<td>41.481</td>
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<tr>
<td>$R_l$</td>
<td>$20.767 \pm 0.025$</td>
<td>20.739</td>
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<tr>
<td>$A_{l,b}^{0,l}$</td>
<td>$0.01714 \pm 0.00095$</td>
<td>0.01642</td>
</tr>
<tr>
<td>$A_{l}(P_o)$</td>
<td>$0.1465 \pm 0.0032$</td>
<td>0.1480</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$0.21630 \pm 0.00066$</td>
<td>0.21562</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$0.1723 \pm 0.0031$</td>
<td>0.1723</td>
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<tr>
<td>$A_{l,b}^{0,b}$</td>
<td>$0.0992 \pm 0.0016$</td>
<td>0.1037</td>
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<tr>
<td>$A_{l,c}^{0,c}$</td>
<td>$0.0707 \pm 0.0035$</td>
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<tr>
<td>$A_b$</td>
<td>$0.923 \pm 0.020$</td>
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<tr>
<td>$A_c$</td>
<td>$0.670 \pm 0.027$</td>
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<tr>
<td>$A_l(SLD)$</td>
<td>$0.1513 \pm 0.0021$</td>
<td>0.1480</td>
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<tr>
<td>$\sin^2 \theta_{\text{eff}}^{\text{lept}} (Q_{fb})$</td>
<td>$0.2324 \pm 0.0012$</td>
<td>0.2314</td>
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<tr>
<td>$m_W$ [GeV]</td>
<td>$80.425 \pm 0.034$</td>
<td>80.390</td>
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<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>$2.133 \pm 0.069$</td>
<td>2.093</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>$178.0 \pm 4.3$</td>
<td>178.4</td>
</tr>
</tbody>
</table>

LEP Electroweak Working Group, Winter 2005
Parity violation in atoms

Nucleon appears elementary at very low $Q^2$; effective Lagrangian for nucleon $\beta$-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}} \bar{e} \gamma_\lambda (1 - \gamma_5) \nu \bar{p} \gamma^\lambda (1 - g_A \gamma_5) n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2 \theta_W$):

$$\mathcal{L}_{ep} = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{p} \gamma^\lambda (1 - 4x_W - \gamma_5) p,$$

$$\mathcal{L}_{en} = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{n} \gamma^\lambda (1 - \gamma_5) n$$

▷ Regard nucleus as a noninteracting collection of $Z$ protons and $N$ neutrons  ▷ Perform NR reduction; nucleons contribute coherently to $A e V_N$ coupling, so dominant $P$-violating contribution to $eN$ amplitude is

$$\mathcal{M}_{pv} = \frac{-iG_F}{2\sqrt{2}} Q^W \bar{e} \rho_N(r) \gamma_5 e$$

$\rho_N(r)$: nucleon density at $e^-$ coordinate $r$

$Q^W \equiv Z(1 - 4x_W) - N$: weak charge

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(Cs) = -72.06 \pm 0.28 \text{ (expt)} \pm 0.34 \text{ (theory)}$$

about 2.5$\sigma$ above SM prediction
The vacuum energy problem

Higgs potential  \( V(\varphi^\dagger \varphi) = \mu^2(\varphi^\dagger \varphi) + |\lambda| (\varphi^\dagger \varphi)^2 \)

At the minimum,

\[
V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.
\]

Identify  \( M_H^2 = -2\mu^2 \)

contributes field-independent vacuum energy density

\[
\rho_H = \frac{M_H^2 v^2}{8}
\]

Adding vacuum energy density  \( \rho_{\text{vac}} \) \( \Leftrightarrow \) adding cosmological constant  \( \Lambda \) to Einstein’s equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}
\]

\[
\Lambda = \frac{8\pi G_N}{c^4} \rho_{\text{vac}}
\]
observed vacuum energy density $\rho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4$

But $M_H \gtrsim 114 \text{ GeV}/c^2 \Rightarrow$

$\rho_H \gtrsim 10^8 \text{ GeV}^4$

MISMATCH BY 54 ORDERS OR MAGNITUDE
Why a Higgs Boson Must Exist

- Role in canceling high-energy divergences

$S$-matrix analysis of $e^+e^- \rightarrow W^+W^-$

\[
\begin{array}{c}
\begin{array}{c}
\text{(a)} \\
\begin{array}{c}
\text{W}^- \\
\gamma \\
\text{e}^- \\
\text{e}^+ \\
\text{W}^+ \\
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\text{(b)} \\
\begin{array}{c}
\text{W}^- \\
\text{Z} \\
\text{e}^- \\
\text{e}^+ \\
\text{W}^+ \\
\end{array}
\end{array}
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{(c)} \\
\begin{array}{c}
\text{W}^- \\
\nu \\
\text{e}^- \\
\text{e}^+ \\
\text{W}^+ \\
\end{array}
\end{array}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\text{(d)} \\
\begin{array}{c}
\text{W}^- \\
\text{H} \\
\text{e}^- \\
\text{e}^+ \\
\text{W}^+ \\
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\]

\( J = 1 \) partial-wave amplitudes \( \mathcal{M}^{(1)}_{\gamma}, \mathcal{M}^{(1)}_{Z}, \mathcal{M}^{(1)}_{\nu} \) have—individually—unacceptable high-energy behavior \( (\propto s) \)
But sum is well-behaved

“Gauge cancellation” observed at LEP2, Tevatron
$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} : \text{ unacceptable HE behavior}$$

(no contributions from $\gamma$ and $Z$)

This divergence is canceled by the Higgs-boson contribution

$$\Rightarrow H e \bar{e} \text{ coupling must be } \propto m_e,$$

because “wrong-helicity” amplitudes $\propto m_e$

If the Higgs boson did not exist, something else would have to cure divergent behavior
IF gauge symmetry were unbroken . . .

▷ no Higgs boson
▷ no longitudinal gauge bosons
▷ no extreme divergences
▷ no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

▷ gauge structure of couplings eliminates the most severe divergences
▷ lesser—but potentially fatal—divergence arises because the electron has mass

. . . due to the Higgs mechanism

▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation must exist in any acceptable theory
**Bounds on $M_H$**

EW theory does not predict Higgs-boson mass

Self-consistency $\Rightarrow$ plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large large energies)—for any $M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad H H / \sqrt{2} \quad H Z_L^0$$

$L$: longitudinal, $1/\sqrt{2}$ for identical particles
In HE limit, \(^s\)-wave amplitudes \(\propto G_F M_H^2 \)

\[
\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi \sqrt{2}} \cdot \begin{bmatrix}
1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\
1/\sqrt{8} & 3/4 & 1/4 & 0 \\
1/\sqrt{8} & 1/4 & 3/4 & 0 \\
0 & 0 & 0 & 1/2
\end{bmatrix}
\]

Require that largest eigenvalue respect the partial-wave unitarity condition \(|a_0| \leq 1\)

\[
\implies M_H \leq \left(\frac{8\pi \sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV}/c^2
\]

condition for perturbative unitarity

\(^a\)Convenient to calculate using Goldstone-boson equivalence theorem, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by \(\mathcal{L}_{\text{int}} = -\lambda v h (2w^+ w^- + z^2 + h^2) - (\lambda/4)(2w^+ w^- + z^2 + h^2)^2\), with \(1/v^2 = G_F \sqrt{2}\) and \(\lambda = G_F M_H^2 / \sqrt{2}\).
If the bound is respected

- weak interactions remain weak at all energies
- perturbation theory is everywhere reliable

If the bound is violated

- perturbation theory breaks down
- weak interactions among $W^\pm$, $Z$, and $H$ become strong on the 1-TeV scale

$\Rightarrow$ features of strong interactions at GeV energies will characterize electroweak gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes $a_{IJ}$ follows from chiral symmetry

\[
\begin{align*}
    a_{00} & \approx \frac{G_F s}{8\pi\sqrt{2}} \quad \text{attractive} \\
    a_{11} & \approx \frac{G_F s}{48\pi\sqrt{2}} \quad \text{attractive} \\
    a_{20} & \approx -\frac{G_F s}{16\pi\sqrt{2}} \quad \text{repulsive}
\end{align*}
\]

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV
Triviality of scalar field theory

Only noninteracting scalar field theories make sense on all energy scales.

Quantum field theory vacuum is a dielectric medium that screens charge \( \Rightarrow \) effective charge is a function of the distance or, equivalently, of the energy scale.

**Running coupling constant**

In \( \lambda \phi^4 \) theory, it is easy to calculate the variation of the coupling constant \( \lambda \) in perturbation theory by summing bubble graphs.

\[
\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log \left( \frac{\Lambda}{\mu} \right)
\]

(Perturbation theory reliable only when \( \lambda \) is small, lattice field theory treats strong-coupling regime)
For stable Higgs potential (i.e., for vacuum energy not to race off to $-\infty$), require $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log (\Lambda/\mu).$$

implies an upper bound

$$\lambda(\mu) \leq \frac{2\pi^2}{3} \log (\Lambda/\mu).$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding $\mu$ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero. $\to$ free field theory “trivial”

Rewrite as bound on $M_H$:

$$\Lambda \leq \mu \exp \left( \frac{2\pi^2}{3\lambda(\mu)} \right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp \left( \frac{4\pi^2 v^2}{3M_H^2} \right)$$
Moral: For any $M_H$, there is a maximum energy scale $\Lambda^*$ at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies.

Perturbative analysis breaks down when $M_H \to 1$ TeV/c$^2$ and interactions become strong.

Lattice analyses $\Rightarrow M_H \lesssim 710 \pm 60$ GeV/c$^2$ if theory describes physics to a few percent up to a few TeV.

If $M_H \to 1$ TeV EW theory lives on brink of instability.
Lower bound by requiring EWSB vacuum

\[ V(v) < V(0) \]

Requiring that \( \langle \phi \rangle_0 \neq 0 \) be an absolute minimum of the one-loop potential up to a scale \( \Lambda \) yields the vacuum-stability condition

\[
M_H^2 > \frac{3G_F \sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\frac{\Lambda^2}{v^2})
\]

\( \ldots \) for \( m_t \approx M_W \)

(No illuminating analytic form for heavy \( m_t \))

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale \( \Lambda^* = 10^{16} \) GeV, then

\[
134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2
\]
Higgs-Boson Properties

\[ \Gamma(H \to f \bar{f}) = \frac{G_F m_f^2 M_H}{4\pi \sqrt{2}} \cdot N_c \cdot \left( 1 - \frac{4m_f^2}{M_H^2} \right)^{3/2} \]

\( \propto M_H \) in the limit of large Higgs mass

\[ \Gamma(H \to W^+ W^-) = \frac{G_F M_H^3}{32\pi \sqrt{2}} (1 - x)^{1/2} (4 - 4x + 3x^2) \]

\( x \equiv 4M_W^2 / M_H^2 \)

\[ \Gamma(H \to Z^0 Z^0) = \frac{G_F M_H^3}{64\pi \sqrt{2}} (1 - x')^{1/2} (4 - 4x' + 3x'^2) \]

\( x' \equiv 4M_Z^2 / M_H^2 \)

asymptotically \( \propto M_H^3 \) and \( \frac{1}{2} M_H^3 \), respectively

(\( \frac{1}{2} \) from weak isospin)

2\( x^2 \) and 2\( x'^2 \) terms \( \Leftrightarrow \) decays into transversely polarized gauge bosons

Dominant decays for large \( M_H \) into pairs of longitudinally polarized weak bosons
Below $W^+W^-$ threshold, $\Gamma_H \ll 1 \text{ GeV}$

Far above $W^+W^-$ threshold, $\Gamma_H \propto M_H^3$

For $M_H \to 1 \text{ TeV}/c^2$, Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.
Clues to the Higgs-boson mass

Sensitivity of EW observables to $m_t$ gave early indications for massive top quantum corrections to SM predictions for $M_W$ and $M_Z$ arise from different quark loops

\[ M_W^2 = M_Z^2 \left( 1 - \sin^2 \theta_W \right) (1 + \Delta \rho) \]

where $\Delta \rho \approx \Delta \rho^{\text{quarks}} = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}}$

strong dependence on $m_t^2$ accounts for precision of $m_t$ estimates derived from EW observables

$m_t$ known to $\pm 3\%$ from Tevatron 

look beyond the quark loops to next most important quantum corrections:

Higgs-boson effects
$H$ quantum corrections smaller than $t$ corrections, exhibit more subtle dependence on $M_H$ than the $m_t^2$ dependence of the top-quark corrections

$$\Delta \rho^{(\text{Higgs})} = C \cdot \ln \left( \frac{M_H}{\mu} \right)$$

$M_Z$ known to 23 ppm, $m_t$ and $M_W$ well measured

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>CDF</strong></td>
<td><strong>TEVATRON</strong></td>
</tr>
<tr>
<td>176.1 ± 6.6</td>
<td>80.452 ± 0.059</td>
</tr>
<tr>
<td><strong>DØ</strong></td>
<td><strong>LEP2</strong></td>
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<td>179.0 ± 5.1</td>
<td>80.412 ± 0.042</td>
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<tr>
<td><strong>Average</strong></td>
<td><strong>Average</strong></td>
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<tr>
<td>178.0 ± 4.3</td>
<td>80.425 ± 0.034</td>
</tr>
<tr>
<td>$\chi^2$/DoF: 2.6 / 4</td>
<td>$\chi^2$/DoF: 0.3 / 1</td>
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<tr>
<td><strong>LEP1/SLD</strong></td>
<td><strong>NuTeV</strong></td>
</tr>
<tr>
<td><strong>172^{+13}_{-10}</strong></td>
<td><strong>80.136 ± 0.084</strong></td>
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<tr>
<td><strong>LEP1/SLD/m_w/G_w</strong></td>
<td><strong>LEP1/SLD</strong></td>
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<td><strong>181^{+12}_{-9}</strong></td>
<td><strong>80.363 ± 0.032</strong></td>
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<tr>
<td><strong>LEP1/SLD/m_t</strong></td>
<td><strong>LEP1/SLD/m_t</strong></td>
</tr>
<tr>
<td><strong>80.452 ± 0.059</strong></td>
<td><strong>80.373 ± 0.023</strong></td>
</tr>
</tbody>
</table>

so examine dependence of $M_W$ upon $m_t$ and $M_H$
Direct, indirect determinations agree reasonably
Both favor a light Higgs boson,

within framework of SM analysis.
Best Tevatron Run II (*Preliminary)

* D0 Dilepton  
(L = 230 pb$^{-1}$)  
$155.0 \pm 14.0 \pm 7.0$

* CDF Dilepton  
(L = 340 pb$^{-1}$)  
$168.3 \pm 5.3 \pm 3.3$

* D0 Lepton+Jets  
(L = 320 pb$^{-1}$)  
$169.5 \pm 3.0 \pm 3.6$

CDF Lepton+Jets  
(L = 318 pb$^{-1}$)  
$173.5 \pm 2.7 \pm 2.8$

* Tevatron EPS 2005  
(CDF+D0 Run I+II Average)  
$172.7 \pm 1.7 \pm 2.4$

Top mass (GeV/c$^2$)
Fit to a universe of data

\[ \Delta \alpha_{\text{had}}^{(5)} = \]

- 0.02758 ± 0.00035
- 0.02749 ± 0.00012

incl. low \( Q^2 \) data

Excluded

\[ \Delta \chi^2 \]

\[ m_H [\text{GeV}] \]
Within SM, LEPEWWG deduce a 95% CL upper limit, $M_H \lesssim 219$ GeV/$c^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/$c^2$, excluding much of the favored region

either the Higgs boson is just around the corner, or

SM analysis is misleading

Things will soon be popping!
Within SM, LEPEWWG deduce a 95% CL upper limit, $M_H \lesssim 219$ GeV/$c^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/$c^2$, excluding much of the favored region.

either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

Expect progress from $M_W$-$m_t$-$M_H$ correlation

▷ Tevatron and LHC measurements will determine $m_t$ within 1 or 2 GeV/$c^2$

▷ ...and improve $\delta M_W$ to about 15 MeV/$c^2$

▷ As the Tevatron’s integrated luminosity approaches 10 fb$^{-1}$, CDF and DØ will begin to explore the region of $M_H$ not excluded by LEP

▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC
Assessment

25 YEARS OF CONFIRMATIONS OF $SU(2)_L \otimes U(1)_Y$

★ neutral currents
★ $W^\pm, Z^0$
★ charm

(+ experimental guidance)

★ $\tau, \nu_\tau$
★ $b, t$

+ experimental surprises
★ narrowness of $\psi, \psi'$
★ long $B$ lifetime
★ large $B^0-\bar{B}^0$ mixing
★ heavy top
★ neutrino oscillations
10 YEARS OF PRECISION MEASUREMENTS... 
... FIND NO SIGNIFICANT DEVIATIONS
QUANTUM CORRECTIONS TESTED AT $\pm 10^{-3}$

NO “NEW” PHYSICS ... YET!

Theory tested at distances
from $10^{-17}$ cm
to $\sim 10^{22}$ cm

origin Coulomb’s law (tabletop experiments)
smaller \begin{align*}
\text{Atomic physics} & \rightarrow \text{QED} \\
\text{high-energy experiments} & \rightarrow \text{EW theory}
\end{align*}
larger $M_\gamma \approx 0$ in planetary ... measurements

IS EW THEORY TRUE ?
COMPLETE ??
EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity: dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions—electrons—and gauge interactions—QED—needed to generate the scalar bound states are already present in the case of superconductivity. Could a scheme of similar economy account for EWSB?

\[ SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d \]

Treat \( SU(2)_L \otimes U(1)_Y \) as perturbation

\[ m_u = m_d = 0: \] QCD has exact \( SU(2)_L \otimes SU(2)_R \) chiral symmetry. At an energy scale \( \sim \Lambda_{QCD} \), strong interactions become strong, fermion condensates appear, and \( SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V \)

\[ \implies 3 \text{ Goldstone bosons, one for each broken generator: } 3 \text{ massless pions (Nambu) } \]
Broken generators: 3 axial currents; couplings to $\pi$ measured by pion decay constant $f_\pi$

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix}
g^2 & 0 & 0 & 0 \\
0 & g^2 & 0 & 0 \\
0 & 0 & g^2 & gg' \\
0 & 0 & gg' & g'^2
\end{pmatrix} \frac{f_\pi^2}{4},$$

$(W^+, W^-, W_3, A)$

Same structure as standard EW theory. Diagonalize:

$M^2_W = g^2 f_\pi^2/4, M^2_Z = (g^2 + g'^2) f_\pi^2/4, M^2_A = 0$, so

$$\frac{M^2_Z}{M^2_W} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons

$$M_W \approx 30 \text{ MeV}/c^2$$