This is the first course at UCSD for which “Lecture on Demand” have been made available. 2000 Minutes of Streaming video served to hundreds of “demands” without interruption (24/7)

Pl. take 10 minutes to fill out the Streaming Video Survey sent to you last week by the Physics Department.

Your input alone will decide whether to extend this educational service to other classes at UCSD and to the next generation of UC students.
The Coulomb Attractive Potential That Binds the electron and Nucleus (charge \( +Ze \)) into a Hydrogenic atom

\[
U(r) = \frac{kZe^2}{r}
\]

The Hydrogen Atom In Its Full Quantum Mechanical Glory

As in case of particle in 3D box, we should use seperation of variables \((x,y,z)\) to derive 3 independent differential eqns. This approach will get very ugly since we have a "conjoined triplet".
To simplify the situation, choose more appropriate variables
Cartesian coordinates \((x,y,z)\) → Spherical Polar \((r, \theta, \phi)\) coordinates

\[
x = r \sin \theta \cos \phi \\
y = r \sin \theta \sin \phi \\
z = r \cos \theta \\
r = \sqrt{x^2 + y^2 + z^2} \\
\theta = \cos^{-1} \frac{z}{r} \quad \text{(Polar angle)} \\
\phi = \tan^{-1} \frac{y}{x} \quad \text{(Azimuthal angle)}
\]
Spherical Polar Coordinate System

Volume Element $dV$

$$dV = (r \sin \theta d\phi)(r d\theta)(dr) = r^2 \sin \theta dr d\theta d\phi$$

The Hydrogen Atom In Its Full Quantum Mechanical Glory

Instead of writing Laplacian $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$,
write $\nabla^2$ for spherical polar coordinates:

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Thus the T.I.S.Eq. for $\psi(x,y,z) = \psi(r,\theta,\phi)$ becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi(r,\theta,\phi)}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi(r,\theta,\phi)}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi(r,\theta,\phi)}{\partial \phi^2} + \frac{2m}{\hbar^2} \left( E-U(r) \right) \psi(r,\theta,\phi) = 0$$

with $U(r) \propto \frac{1}{r} = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$
The Schrödinger Equation in Spherical Polar Coordinates (is bit of a mess!)

The TISE is:
\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial^2 \phi} + \frac{2m}{\hbar^2} (E-U(r)) \psi(r, \theta, \phi) = 0
\]

Try to free up second last term from all except \( \phi \)

This requires multiplying throughout by \( r^2 \sin^2 \theta \) \( \Rightarrow \)

\[
\sin^2 \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{\partial^2 \psi}{\partial^2 \phi} + \frac{2mr^2 \sin^2 \theta}{\hbar^2} (E+\frac{ke^2}{r}) \psi = 0
\]

For Separation of Variables, Write \( \psi(r, \theta, \phi) = R(r) \cdot \Theta(\theta) \cdot \Phi(\phi) \)

Plug it into the TISE above & divide throughout by \( \psi(r, \theta, \phi) = R(r) \Theta(\theta) \Phi(\phi) \)

<table>
<thead>
<tr>
<th>Term</th>
<th>Separating Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(r) )</td>
<td>( \frac{\partial \psi(r, \theta, \phi)}{\partial r} = \Theta(\theta) \Phi(\phi) \frac{\partial R(r)}{\partial r} )</td>
</tr>
<tr>
<td>( R(r) \Theta(\theta) )</td>
<td>( \frac{\partial \psi(r, \theta, \phi)}{\partial \theta} = R(r) \Phi(\phi) \frac{\partial \Theta(\theta)}{\partial \theta} )</td>
</tr>
<tr>
<td>( R(r) \Theta(\theta) \Phi(\phi) )</td>
<td>( \frac{\partial \psi(r, \theta, \phi)}{\partial \phi} = R(r) \Theta(\theta) \frac{\partial \Phi(\phi)}{\partial \phi} )</td>
</tr>
</tbody>
</table>

Note that:

\[
\frac{\partial^2 \psi(r, \theta, \phi)}{\partial^2 \phi} = \frac{\partial \Theta(\theta)}{\partial \theta} \frac{\partial \Phi(\phi)}{\partial \phi} \Rightarrow \text{when substituted in TISE}
\]

\[
\sin^2 \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial R}{\partial r} \right) + \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Theta}{\partial \theta} \right) + \frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial^2 \phi} + \frac{2mr^2 \sin^2 \theta}{\hbar^2} (E+\frac{ke^2}{r}) = 0
\]

Rearrange by taking the \( \phi \) term on RHS

\[
\sin^2 \theta \frac{\partial}{\partial r} \left( r^2 \frac{\partial R}{\partial r} \right) + \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Theta}{\partial \theta} \right) + \frac{2mr^2 \sin^2 \theta}{\hbar^2} (E+\frac{ke^2}{r}) = -\frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial^2 \phi}
\]

LHS is fn. of \( r, \theta \) & RHS is fn of \( \phi \) only, for equality to be true for all \( r, \theta, \phi \)

\[
\Rightarrow \text{LHS} = \text{constant} = \text{RHS} = \text{m}_i^2
\]
Deconstructing The Schrodinger Equation for Hydrogen

Now go break up LHS to separate the $r$ & $\theta$ terms.....

LHS: \[ \frac{\sin^2 \theta}{R} \frac{\partial}{\partial r} \left( r^2 \frac{\partial R}{\partial r} \right) + \frac{\sin \theta}{\Theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Theta}{\partial \theta} \right) + \frac{2mr^2}{\hbar^2} \left( E + \frac{ke^2}{r} \right) = m_i^2 \]

Divide Thruout by $\sin^2 \theta$ and arrange all terms with $r$ away from $\theta \Rightarrow$

\[ \frac{1}{R} \frac{\partial}{\partial r} \left( r^2 \frac{\partial R}{\partial r} \right) + \frac{2mr^2}{\hbar^2} (E + \frac{ke^2}{r}) = \frac{m_i^2}{\sin^2 \theta} - \frac{1}{\Theta \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Theta}{\partial \theta} \right) \]

Same argument : LHS is fn of $r$, RHS is fn of $\theta$;

For them to be equal for all $r, \theta \Rightarrow \text{LHS = const} = \text{RHS} = l(l+1)$

What is the mysterious $l(l+1)$? Just a number like $2(2+1)$

So What do we have after all the shuffling!

\[ \frac{d^2 \Phi}{d\phi^2} + m_i^2 \Phi = 0 \ldots \ldots \ldots \ldots \ldots (1) \]
\[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d \Theta}{d\theta} \right) + \left[ l(l+1) - \frac{m_i^2}{\sin^2 \theta} \right] \Theta(\theta) = 0 \ldots \ldots \ldots \ldots \ldots (2) \]
\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) + \left[ \frac{2mr^2}{\hbar^2} \left( E + \frac{ke^2}{r} \right) \frac{l(l+1)}{r^2} \right] R(r) = 0 \ldots \ldots \ldots \ldots \ldots (3) \]

These 3 "simple" diff. eqn describe the physics of the Hydrogen atom.

All we need to do now is guess the solutions of the diff. equations

Each of them, clearly, has a different functional form
And Now the Solutions of The S. Eqns for Hydrogen Atom

The Azimuthal Diff. Equation:
\[ \frac{d^2 \Phi}{d\phi^2} + \frac{m^2 \Phi}{\ell (\ell + 1)} = 0 \]

Solution: \( \Phi(\phi) = A e^{im\phi} \) but need to check "Good Wavefunction Condition"

Wave Function must be Single Valued for all \( \phi \) \( \Rightarrow \Phi(\phi) = \Phi(\phi + 2\pi) \)

\( \Rightarrow \Phi(\phi) = A e^{im\phi} = A e^{im(\phi + 2\pi)} \Rightarrow m = 0, \pm 1, \pm 2, \ldots \) (Magnetic Quantum #)

The Polar Diff. Eq:
\[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \left[ \ell (\ell + 1) - \frac{m^2}{\sin^2 \theta} \right] \Theta(\theta) = 0 \]

Solutions: go by the name of "Associated Legendre Functions"
only exist when the integers \( l \) and \( m \) are related as follows
\[ m = 0, \pm 1, \pm 2, \pm 3, \ldots \pm l \] ; \( l = \text{positive number} \)

\[ l : \text{Orbital Quantum Number} \]

Wavefunction Along Azimuthal Angle \( \phi \) and Polar Angle \( \theta \)

For \( l = 0, m = 0 \) \( \Rightarrow \Theta(\theta) = \frac{1}{\sqrt{2}} \)

For \( l = 1, m = 0, \pm 1 \) \( \Rightarrow \text{Three Possibilities for the Orbital part of wavefunction} \)

\[ [l = 1, m = 0] \Rightarrow \Theta(\theta) = \frac{\sqrt{6}}{2} \cos \theta \]

\[ [l = 1, m = \pm 1] \Rightarrow \Theta(\theta) = \frac{\sqrt{3}}{2} \sin \theta \]

\[ [l = 2, m = 0] \Rightarrow \Theta(\theta) = \frac{\sqrt{10}}{4} (3\cos^2 \theta - 1) \]

\( ... \text{and so on and so forth} \) (see book for more Functions)
Radial Differential Equations and Its Solutions

The Radial Diff. Eqn: \[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{\partial R}{\partial r} \right) + \left[ \frac{2m_e}{\hbar^2} (E + \frac{ke^2}{r}) - \frac{l(l + 1)}{r^2} \right] R(r) = 0 \]

_Solutions_: Associated Laguerre Functions \( R(r) \), Solutions exist only if:

1. \( E > 0 \) or has negative values given by
   \[ E = -\frac{ke^2}{2a_0} \left( \frac{1}{n^2} \right) \]
   \( a_0 = \frac{\hbar^2}{me^2} = \) Bohr Radius

2. And when \( n = \) integer such that \( l = 0, 1, 2, 3, 4, ..., (n-1) \)
   \[ n = \text{principal Quantum } \# \text{ or the "big daddy" quantum } \# \]

The Hydrogen Wavefunction: \( \psi(r, \theta, \phi) \) and \( \Psi(r, \theta, \phi, t) \)

To Summarize: The hydrogen atom is brought to you by the letters:

\[
\begin{align*}
\begin{array}{c}
n = 1, 2, 3, 4, 5, \ldots, \infty \\
l = 0, 1, 2, 3, \ldots, (n-1) \\
m_l = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm l \\
\end{array}
\end{align*}
\]

Quantum \# appear only in Trapped systems

The Spatial part of the Hydrogen Atom Wave Function is:

\[
\psi(r, \theta, \phi) = R_{nl}(r) \cdot \Theta_{lm_l}(\theta) \cdot \Phi_{m_l}(\phi) = R_{nl} Y_{l}^{m_l}
\]

\( Y_{l}^{m_l} \) are known as Spherical Harmonics. They define the angular structure in the Hydrogen-like atoms.

The Full wavefunction is \( \Psi(r, \theta, \phi, t) = \psi(r, \theta, \phi)e^{-\frac{iEt}{\hbar}} \)
Radial Wave Functions For n=1,2,3

\[
R(r) = \begin{cases} 
\frac{2}{a_0^{3/2}} e^{-\frac{r}{a_0}} & \text{for } n=1, m_l=0, m_m=0 \\
\frac{1}{2\sqrt{2}a_0^{3/2}}(2-\frac{r}{a_0})e^{-\frac{r}{2a_0}} & \text{for } n=2, m_l=0, m_m=0 \\
\frac{2}{84\sqrt{3}a_0^{3/2}}(27-18\frac{r}{a_0}+2\frac{r^2}{a_0^2})e^{-\frac{r}{3a_0}} & \text{for } n=3, m_l=0, m_m=0 
\end{cases}
\]

- n=1 → K shell
- n=2 → L Shell
- n=3 → M shell
- n=4 → N Shell
- .......

Symbolic Notation of Atomic States in Hydrogen

\[
l \rightarrow \ s \ (l = 0) \quad p \ (l = 1) \quad d \ (l = 2) \quad f \ (l = 3) \quad g \ (l = 4) \quad .......
\]

\[
n \downarrow
\]

1 \quad 1s
2 \quad 2s \quad 2p
3 \quad 3s \quad 3p \quad 3d
4 \quad 4s \quad 4p \quad 4d \quad 4f
5 \quad 5s \quad 5p \quad 5d \quad 5f \quad 5g
\]

Note that:
- \( n = 1 \) is a non-degenerate system
- \( n > 1 \) are all degenerate in \( l \) and \( m_l \)
  All states have same energy
  But different angular configuration

\[
E = -\frac{ke^2}{2a_0}\left(\frac{1}{n^2}\right)
\]
Energy States, Degeneracy & Transitions

\[ n, l, m = 0 \Rightarrow R(r) = \frac{2}{a_0^2} e^{-a_0 r}; \quad \Theta(\theta) = \frac{1}{\sqrt{2\pi}}; \quad \Phi(\phi) = \frac{1}{\sqrt{2}} \]

\[ \Psi_{100}(r, \theta, \phi) = \frac{1}{a_0 \sqrt{\pi}} e^{a_0 r} \] .......look at it carefully

1. Spherically symmetric \( \Rightarrow \) no \( \theta, \phi \) dependence (structure)

2. Probability Per Unit Volume: \( |\Psi_{100}(r, \theta, \phi)|^2 = \frac{1}{\pi a_0} e^{-\frac{2r}{a_0}} \)

Likelihood of finding the electron is same at all \( \theta, \phi \) and depends only on the radial separation \( r \) between electron & the nucleus.

3 Energy of Ground State = \( \frac{-ke^2}{2a_0} = -13.6 eV \)

Overall The Ground state wavefunction of the hydrogen atom is quite \textit{boring}

Not much chemistry or Biology could develop if there was only the ground state of the Hydrogen Atom!

\textbf{We need structure, we need variety, we need some curves!}
**Interpreting Orbital Quantum Number \((l)\)**

Radial part of S.Eqn: 
\[
\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left[ \frac{2m}{\hbar^2} \left( \frac{ke^2}{r} - \frac{l(l+1)}{r^3} \right) \right] R(r) = 0
\]

For H Atom:  
\[
E = K + U = K_{\text{RADIAL}} + K_{\text{ORBITAL}} + \frac{ke^2}{r} \]
substitute this in \(E\)

\[
\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left[ K_{\text{RADIAL}} + K_{\text{ORBITAL}} \right] \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} R(r) = 0
\]

Examine the equation, if we set \(K_{\text{ORBITAL}} = \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2}\) then

what remains is a differential equation in \(r\)

\[
\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left[ K_{\text{RADIAL}} \right] R(r) = 0 \text{ which depends only on radius } r \text{ of orbit}
\]

Further, we also know that \(K_{\text{ORBITAL}} = \frac{1}{2} m v^2 \leftrightarrow \mathbf{L} = \mathbf{\hat{r}} \times \mathbf{p} \);  
\(|L| = m v r \Rightarrow K_{\text{ORBITAL}} = \frac{L^2}{2m^2 r^2}\)

Putting it all together: \(K_{\text{ORBITAL}} = \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} = \frac{L^2}{2m^2 r^2} \Rightarrow \text{magnitude of Ang. Mom} |L| = \sqrt{l(l+1)} \hbar\)

Since \(l = \text{positive integer} = 0,1,2,3\ldots (n-1) \Rightarrow \text{angular momentum} |L| = \sqrt{l(l+1)} \hbar = \text{discrete values}\)

| \(L| = \sqrt{l(l+1)} \hbar \);  \text{ QUANTIZATION OF Electron's Angular Momentum} |

---

**Magnetic Quantum Number \(m_l\)**

\(\mathbf{\bar{L}} = \mathbf{\hat{r}} \times \mathbf{\bar{p}} \) (Right Hand Rule)

Classically, direction & Magnitude of \(\mathbf{\bar{L}}\) always well defined

QM: Can/Does \(\mathbf{\bar{L}}\) have a definite direction? Proof by Negation:

Suppose \(\mathbf{\bar{L}}\) was precisely known/defined \((\mathbf{\bar{L}} \parallel \mathbf{\hat{z}})\)

Since \(\mathbf{\bar{L}} = \mathbf{\hat{r}} \times \mathbf{\bar{p}} \Rightarrow \text{Electron MUST be in x-y orbit plane}\)

\(\Rightarrow \Delta z = 0 ; \Delta p_z \Delta z \approx \hbar \Rightarrow \Delta p_z \sim \infty ; \ E = \frac{p^2}{2m} \sim \infty \ !!!\)

So, in Hydrogen atom, \(\mathbf{\bar{L}}\) cannot have precise measurable value

Uncertainty Principle & Angular Momentum: \(\Delta L_z \Delta \phi \sim \hbar\)
Magnetic Quantum Number \( m_l \)

Consider \( \ell = 2 \nabla L = \sqrt{\ell(\ell+1)} = \sqrt{6}\hbar \)

In Hydrogen atom, \( \vec{L} \) can not have precise measurable value

Arbitrarily picking Z axis as a reference direction:

\( \vec{L} \) vector spins around Z axis (precesses).

The Z component of \( \vec{L} \): \( |L_z| = m_l\hbar; m_l = \pm 1, \pm 2, \pm 3... \pm \ell \)

Note: since \( |L_z| < |L| \) (always)

since \( m_l\hbar < \sqrt{\ell(\ell+1)}\hbar \) It can never be that \( |L_z| = m_l\hbar = \sqrt{\ell(\ell+1)}\hbar \)

(breaks Uncertainty Principle)

So......the Electron's dance has begun!

\( L=2, m_l=0, \pm 1, \pm 2 \): Pictorially

\[ L = \hbar \sqrt{\ell(\ell+1)} = \hbar \sqrt{2(2+1)} = \hbar \sqrt{6} \]

Electron "sweeps"

Conical paths of different \( \Theta \): \( \cos \Theta = L_z/L \)

On average, the angular momentum Component in x and y cancel out

\( \langle L_x \rangle = 0 \)
\( \langle L_y \rangle = 0 \)
Where is it likely to be? \( \rightarrow \) Radial Probability Densities

\[ \Psi(r, \theta, \phi) = R_n l (r) \cdot \Theta_{m_l} (\theta) \cdot \Phi_{m_m} (\phi) = R_n l Y_{l m}^m \]

Probability Density Function in 3D:

\[ P(r, \theta, \phi) = \Psi^* \Psi = | \Psi(r, \theta, \phi) |^2 = | R_n l |^2 \cdot | Y_{l m}^m |^2 \]

Note: 3D Volume element \( dV = r^2 \sin \theta \, dr \, d\theta \, d\phi \)

Prob. of finding particle in a tiny volume \( dV \) is

\[ P \, dV = | R_n l |^2 \cdot | Y_{l m}^m |^2 \cdot r^2 \sin \theta \, dr \, d\theta \, d\phi \]

The Radial part of Prob. distribution: \( P(r)dr \)

\[ P(r)dr = | R_n l |^2 \cdot r^2 dr \int_0^\pi | \Theta_{m_l} (\theta) |^2 d\theta \int_0^{2\pi} | \Phi_{m_m} (\phi) |^2 d\phi \]

When \( \Theta_{m_l} (\theta) \) & \( \Phi_{m_m} (\phi) \) are auto-normalized then

\[ P(r)dr = | R_n l |^2 \cdot r^2 dr \quad \text{in other words} \quad P(r) = r^2 | R_n l |^2 \]

Normalization Condition:

\[ 1 = \int_0^\infty r^2 | R_n l |^2 dr \]

Expectation Values

\[ \langle f(r) \rangle = \int_0^\infty f(r) \cdot P(r)dr \]

Ground State: Radial Probability Density

\[ P(r)dr = | \psi(r) |^2 \cdot 4\pi r^2 dr \]

\[ \Rightarrow P(r)dr = \frac{4}{a_0^3} r^2 e^{-\frac{r}{a_0}} \]

Probability of finding Electron for \( r > a_0 \)

\[ P_{r>a_0} = \int_a^\infty \frac{4}{a_0^3} r^2 e^{-\frac{r}{a_0}} dr \]

To solve, employ change of variable

Define \( z = \frac{2r}{a_0} \); \textit{change} limits of integration

\[ P_{r=a_0} = \frac{1}{2} \int_a^\infty \frac{z^2 e^{-z}}{a_0} dz \quad \text{(such integrals called Error. Fn)} \]

\[ = \frac{1}{2} [ z^2 + 2z + 2 ] e^{-z} \bigg|_a = 5e^2 = 0.667 \Rightarrow 66.7\% \]
Most Probable & Average Distance of Electron from Nucleus

Most Probable Distance:

In the ground state \((n=1, l=0, m_l=0)\) \(P(r)dr = \frac{4}{a_0^3} r^2 e^{-\frac{2r}{a_0}} dr\)

Most probable distance \(r\) from Nucleus \(\Rightarrow\) What value of \(r\) is \(P(r)\) max?

\[
\frac{dP}{dr} = 0 \quad \Rightarrow \quad \frac{d}{dr} \left[ r^2 e^{-\frac{2r}{a_0}} \right] = 0 \quad \Rightarrow \quad \left[ \frac{-2r^2}{a_0} + 2r \right] e^{-\frac{2r}{a_0}} = 0
\]

\[
\Rightarrow \quad \frac{2r^2}{a_0} + 2r = 0 \quad \Rightarrow \quad r = 0 \quad \text{or} \quad r = a_0 \quad \text{... which solution is correct?}
\]

(see past quiz) : Can the electron BE at the center of Nucleus \((r=0)\)?

\(P(r=0) = \frac{4}{a_0^3} \cdot 0^2 e^{-\frac{0}{a_0}} = 0! \Rightarrow \text{Most Probable distance } r = a_0 \) (Bohr guessed right)

What about the AVERAGE location \(<r>\) of the electron in Ground state?

\(<r> = \int_0^\infty rP(r)dr = \frac{4}{a_0^3} \int_0^\infty r^2 e^{-\frac{2r}{a_0}} dr \quad \text{...change of variable } z = \frac{2r}{a_0}\)

\[
\Rightarrow <r> = \frac{a_0^2}{4} \int_0^\infty z^2 e^{-z}dz \quad \text{...Use general form } \int_0^\infty z^n e^{-z}dz = n! = n(n-1)(n-2)...(1)
\]

\[
\Rightarrow \quad <r> = \frac{a_0^2}{4} 3! = \frac{3a_0^2}{2} \neq a_0 \quad \text{Average & most likely distance is not same. Why?}
\]

Answer is in the form of the radial Prob. Density: Not symmetric

Radial Probability Distribution \(P(r) = r^2 R(r)\)

Because \(P(r) = r^2 R(r)\)

No matter what \(R(r)\) is for some \(n\)

The prob. Of finding electron inside nucleus = 0

Table 7-2: Radial functions for hydrogen

<table>
<thead>
<tr>
<th>(n)</th>
<th>(l)</th>
<th>(R_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>(\frac{2}{a_0} e^{-\frac{r}{a_0}})</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(\frac{1}{2a_0} \left( 1 - \frac{2r}{a_0} \right) e^{-\frac{r}{a_0}})</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(\frac{1}{2a_0} e^{-\frac{r}{a_0}})</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>(\frac{2}{3a_0} \left( 1 - \frac{2r}{a_0} + \frac{2r^2}{27a_0^2} \right) e^{-\frac{r}{a_0}})</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(\frac{2}{27a_0^2} e^{-\frac{r}{a_0}})</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(\frac{4}{81a_0^3} e^{-\frac{r}{a_0}})</td>
</tr>
</tbody>
</table>