Brian Wecht, the TA, is away this week. I will substitute for his office hours (in my office 3314 Mayer Hall, discussion and PS session.

Pl. give all regrade requests to me this week

Quiz 3 is This Friday
Ch 2 : Quantum Theory Of Light

• What is the nature of light ?
  – When it propagates ?
  – When it interacts with Matter?

• What is Nature of Matter ?
  – When it interacts with light ?
  – As it propagates ?

• Revolution in Scientific Thought
  – Like a firestorm of new ideas (every body goes nuts!..not like Evolution)
    • Old concepts violently demolished , new ideas born
      – Interplay of experimental findings & scientific reason

• One such revolution happened at the turn of 20th Century
  – Led to the birth of Quantum Theory & Modern Physics
Classical Picture of Light : Maxwell's Equations

- Maxwell’s Equations:

\[
\oint E \cdot dA = \frac{Q}{\varepsilon_0}
\]
\[
\oint B \cdot dA = 0
\]
\[
\oint E \cdot ds = -\frac{d\Phi_E}{dt}
\]
\[
\oint B \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\]
\[
\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}
\]
\[
\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}
\]

\[
E = E_{\text{max}} \cos(kx - \omega t)
\]
\[
B = B_{\text{max}} \cos(kx - \omega t)
\]

\[
c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}
\]
Properties of EM Waves: Maxwell’s Equations

Energy Flow in EM Waves:

Poynting Vector $\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$

Power incident on an area $A$

$= \vec{S} \cdot \vec{A} = \frac{1}{\mu_0} \left( AE_0 B_0 \sin^2(kx - \omega t) \right)$

Intensity of Radiation $I = \frac{1}{2\mu_0 c} E_0^2$

Larger the amplitude of Oscillation

More intense is the radiation

Disasters in Classical Physics (1899-1922)

• Disaster ➔ Experimental observation that could not be explained by Classical theory (Phys 2A, 2B, 2C)
  
  – Disaster # 1: Nature of Blackbody Radiation from your BBQ grill
  
  – Disaster # 2: Photo Electric Effect
  
  – Disaster # 3: Scattering light off electrons (Compton Effect)

• Resolution of Experimental Observation will require radical changes in how we think about nature
  
  – ➔ QUANTUM MECHANICS
    
    • The Art of Conversation with Subatomic Particles
Nature of Radiation: An Expt with BBQ Grill

Question: Distribution of Intensity of EM radiation Vs T & λ

- Radiator (grill) at some temp T
- Emits variety of wavelengths
  - Some with more intensity than others
- EM waves of diff. λ bend differently within prism
- Eventually recorded by a detector (eye)
- Map out emitted Power / area Vs λ

![Diagram showing the process of radiation from a grill through a prism to a detector]

Notice shape of each curve and learn from it

Radiation from A Blackbody

![Graph showing the intensity R(λ) of radiation from blackbodies at different temperatures]
(a) Intensity of Radiation \( I = \int R(\lambda) d\lambda \propto T^4 \)
\[ I = \sigma T^4 \text{(Area under curve)} \]
Stephan-Boltzmann Constant \( \sigma = 5.67 \times 10^{-8} \text{ W} / \text{m}^2 \text{K}^4 \)

(b) Higher the temperature of BBQ
Lower is the \( \lambda \) of PEAK intensity
\[ \lambda_{\text{MAX}} \propto 1 / T \]
Wein's Law \( \lambda_{\text{MAX}} T = \text{const} = 2.898 \times 10^{-3} \text{ mK} \)
As a body gets hotter it gets more RED then White

Reason for different shape of \( R(\lambda) \) Vs \( \lambda \) for different temperature?
Can one explain in on basis of Classical Physics (2A, 2B, 2C) ??

Blackbody Radiator: An Idealization

Classical Analysis:
- Box is filled with EM standing waves
- Radiation reflected back-and-forth between walls
- Radiation in thermal equilibrium with walls of Box
- How many waves of wavelength \( \lambda \) can fit inside the box ?

Blackbody Absorbs everything
Reflects nothing
All light entering opening gets absorbed (ultimately) by the cavity wall

Cavity in equilibrium \( T \)
w.r.t. surrounding. So it radiates everything it absorbs

Emerging radiation is a sample of radiation inside box at temp \( T \)

Predict nature of radiation inside Box ?
Standing Waves

Classical Calculation

# of standing waves between Wavelengths $\lambda$ and $\lambda + d\lambda$ are:

$$N(\lambda)d\lambda = \frac{8\pi V}{\lambda^4} \cdot d\lambda; V = \text{Volume of box} = L^3$$

Each standing wave contributes energy $E = kT$ to radiation in Box

Energy density $u(\lambda) = \frac{\# \text{ of standing waves/volume}}{} \times \text{Energy/Standing Wave}$

$$u(\lambda) = \frac{8\pi V}{\lambda^4} \times \frac{1}{V} \times kT = \frac{8\pi}{\lambda^4} kT$$

$$R(\lambda) = \frac{4}{4} \frac{8\pi}{\lambda^4} kT = \frac{2\pi c}{\lambda^4} kT$$

Radiancy is Radiation intensity per unit $\lambda$ interval: Let's plot it

**Prediction**: as $\lambda \to 0$ (high frequency) $\Rightarrow R(\lambda) \to \text{Infinity !}$

Oops!
Ultra Violet (Frequency) Catastrophe

Radiancy $R(\lambda)$

- Rayleigh-Jeans law
- Classical Theory
- Experimental Data
- Disaster #1

OOPS!

That was a Disaster! (#1)
Photo Electric Effect: Measurable Properties

- Rate of electron emission from cathode
  - From current $i$ seen in ammeter

- Maximum kinetic energy of emitted electron
  - By applying retarding potential on electron moving towards Collector plate
    - $K_{\text{MAX}} = eV_S$ ($V_S = \text{Stopping voltage}$)
    - Stopping voltage $\rightarrow$ no current flows

- Effect of different types of photo-cathode metal
- Time between shining light and first sign of photo-current in the circuit
Observations: Current Vs Frequency of Incident Light

\[ I_3 = 3I_1 \]
\[ I_2 = 2I_1 \]
\[ I_1 = \text{intensity} \]

Stopping Voltage Vs Incident Light Frequency

Different Metal Photocathode surfaces

\[ eV_s \]
\[ \text{Stopping Voltage} \]
\[ \text{Light frequency} \]
\[ \text{Slope} = \frac{e}{\phi} \]
\[ \text{Intercept} = -\phi \]
Conclusions from the Experimental Observation

- Max Kinetic energy $K_{\text{MAX}}$ independent of Intensity $I$ for light of same frequency.
- No photoelectric effect occurs if light frequency $f$ is below a threshold no matter how high the intensity of light.
- For a particular metal, light with $f > f_0$ causes photoelectric effect IRRESPECTIVE of light intensity.
  - $f_0$ is characteristic of that metal.
- Photoelectric effect is instantaneous!...not time delay.

Can one Explain all this Classically!
As light intensity increased ⇒ \( \overrightarrow{E} \) field amplitude larger
- \( \overrightarrow{E} \) field and electrical force seen by the “charged subatomic oscillators” Larger
  - \( \overrightarrow{F} = e \overrightarrow{E} \)
  - More force acting on the subatomic charged oscillator
  - ⇒ More energy transferred to it
  - ⇒ Charged particle “hooked to the atom” should leave the surface with more Kinetic Energy KE !! The intensity of light shining rules !

As long as light is intense enough, light of ANY frequency \( f \) should cause photoelectric effect

Because the Energy in a Wave is uniformly distributed over the Spherical wavefront incident on cathode, should be a noticeable time lag \( \Delta T \) between time is incident & the time a photo-electron is ejected : Energy absorption time
  - How much time ? Lets calculate it classically.

Classical Physics: Time Lag in Photo-Electric Effect

- Electron absorbs energy incident on a surface area where the electron is confined \( \cong \) size of atom in cathode metal
- Electron is “bound” by attractive Coulomb force in the atom, so it must absorb a minimum amount of radiation before its stripped off
- Example : Laser light Intensity \( I = 120W/m^2 \) on Na metal
  - Binding energy = 2.3 eV = “Work Function”
  - Electron confined in Na atom, size \( \cong 0.1nm \) ..how long before ejection ?
  - Average Power Delivered \( P_{AV} = I \cdot A \). \( A = \pi r^2 \cong 3.1 \times 10^{-20} m^2 \)
  - If all energy absorbed then \( \Delta E = P_{AV} \cdot \Delta T \Rightarrow \Delta T = \Delta E / P_{AV} \)

\[
\Delta T = \frac{(2.3eV)(1.6 \times 10^{-19} J / eV)}{(120W / m^2)(3.1 \times 10^{-20} m^2)} = 0.10 s
\]

- Classical Physics predicts Measurable delay even by the primitive clocks of 1900
- But in experiment, the effect was observed to be instantaneous !!
- Classical Physics fails in explaining all results
Disaster # 2!

Now we need a Hero with New Ideas → Modern Physics!

Max Planck & Birth of Quantum Physics

Back to Blackbody Radiation Discrepancy

Planck noted the UltraViolet Catastrophe at high frequency

“Cooked” calculation with new “ideas” so as bring:

\[ R(\lambda) \to 0 \text{ as } \lambda \to 0 \]
\[ f \to \infty \]

- Cavity radiation as equilibrium exchange of energy between EM radiation & “atomic” oscillators present on walls of cavity
- Oscillators can have any frequency \( f \)
- But the Energy exchange between radiation and oscillator NOT continuous and arbitrary…it is discrete …in packets of same amount
- \[ E = n \ h \ f \text{, with } n = 1, 2, 3, \ldots \infty \]
- \( h \) = constant he invented, a very small number he made up
Planck’s “Charged Oscillators” in a Black Body Cavity
Planck did not know about electrons, Nucleus etc:
They were not known

Planck, Quantization of Energy & BB Radiation

- Keep the rule of counting how many waves fit in a BB Volume
- Radiation Energy in cavity is quantized
- EM standing waves of frequency $f$ have energy
  - $E = nhf$ ($n = 1, 2, 3, \ldots 10, \ldots 1000, \ldots$)
- Probability Distribution: At an equilibrium temp $T$, possible Energy of wave is distributed over a spectrum of states: $P(E) = e^{-E/kT}$
- Modes of Oscillation with:
  - Less energy $E=hf$ = favored
  - More energy $E=hf$ = disfavored

By this statistics, large energy, high $f$ modes of EM disfavored
Planck’s Calculation

\[ R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) \left[ \frac{hc}{\lambda} \left( \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right) \right] \]

Odd looking form

When \( \lambda \rightarrow \text{large} \Rightarrow \frac{hc}{\lambda kT} \rightarrow \text{small} \)

Recall \( e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \)

\[ \Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = (1 + \frac{hc}{\lambda kT} + \frac{1}{2} \left( \frac{hc}{\lambda kT} \right)^2 + \ldots) - 1 = \frac{hc}{\lambda kT} \]

plugging this in \( R(\lambda) \) eqn:

\[ R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) \frac{hc}{\lambda kT} \]

Graph & Compare
With BBQ data

Planck’s Formula and Small \( \lambda \)

When \( \lambda \) is small (large f)

\[ \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \approx \frac{1}{e^{\frac{hc}{\lambda kT}}} \times e^{\frac{hc}{\lambda kT}} = e^{-\frac{hc}{\lambda kT}} \]

Substituting in \( R(\lambda) \) eqn:

\[ R(\lambda) = \left( \frac{c}{4} \right) \left( \frac{8\pi}{\lambda^4} \right) e^{-\frac{hc}{\lambda kT}} \]

As \( \lambda \rightarrow 0 \), \( e^{-\frac{hc}{\lambda kT}} \rightarrow 0 \)

\[ \Rightarrow R(\lambda) \rightarrow 0 \]

Just as seen in the experimental data
Planck’s Explanation of BB Radiation

Fit formula to Exptal data
\[ h = 6.56 \times 10^{-34} \text{ J.S} \]
= very very small

Consequence of Planck’s Formula

Quantization of Energy!