Quiz 1 is This Friday

Quiz 1 will cover Sections 1.1-1.6 (inclusive)
Remaining material will be carried over to Quiz 2

Bring Blue Book, check calculator battery

Write all answers in indelible ink else no grade!

Write answers on consecutive pages: Don’t leave blank pages

All nighters don’t work for these quizzes
Get plenty of Sleep….come with a fresh mind!

Physics 2D Lecture Slides
Lecture 7: Jan 14th 2004

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Relativistic Force & Acceleration

\[ \vec{p} = \frac{m \vec{u}}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} = \gamma m \vec{u} \]

Relativistic Force and Acceleration

\[ \vec{F} = \frac{d \vec{p}}{dt} = \frac{d}{dt} \left( \frac{m \vec{u}}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} \right) \]

\[ F = \frac{m}{\left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2}} \frac{du}{dt} \] : Relativistic Force

Since Acceleration \( \ddot{a} = \frac{d \ddot{u}}{dt} \), [rate of change of velocity]

\[ \ddot{a} = \frac{\vec{F}}{m} \left[1 - \left(\frac{u}{c}\right)^2\right]^{3/2} \]

Reason why you can't quite get up to the speed of light no matter how hard you try!

Note: As \( \frac{u}{c} \to 1 \), \( \ddot{a} \to 0 \)!!!!

It's harder to accelerate when you get closer to speed of light.

Linear Particle Accelerator: Parallel Plates With Potential Difference

\[ \vec{E} = \frac{V}{d} \]

Under force, work is done on the particle, it gains kinetic energy.

New Unit of Energy

1 eV = 1.6 \times 10^{-19} \text{ Joules}
1 MeV = 1.6 \times 10^{-13} \text{ Joules}
1 GeV = 1.6 \times 10^{-10} \text{ Joules}

Charged particle \( q \) moves in straight line in a uniform electric field \( \vec{E} \) with speed \( \vec{u} \) accelerates under force \( \vec{F} = q \vec{E} \)

\[ |\ddot{a}| = \left| \frac{d \ddot{u}}{dt} \right| = \left| \frac{\vec{F}}{m} \right| \left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2} = \left| \frac{q \vec{E}}{m} \right| \left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2} \]

larger the potential difference \( V \) across plates, larger the force on particle.
Linear Particle Accelerator: 50 GigaVolts Accelrating Potential

\[ \ddot{a} = \frac{e \dot{E}}{m} \left[ 1 - \left( \frac{u}{c} \right)^2 \right]^{3/2} \]

Both Rings Housed in Current PEP Tunnel

Magnetic Confinement & Circular Particle Accelerator

Classically

\[ F = m \frac{v^2}{r} \]
\[ qvB = m \frac{v^2}{r} \]

\[ F = \frac{dp}{dt} = \frac{d(\gamma mu)}{dt} = \gamma m \frac{du}{dt} = quB \]

\[ \frac{du}{dt} = \frac{u^2}{r} \quad \text{(Centripetal acceleration)} \]

\[ \gamma m \frac{u^2}{r} = quB \quad \Rightarrow \gamma mu = qBr \quad \Rightarrow p = qBr \]
Charged Form of Matter & Anti-Matter in a B Field

Antimatter form of electron = Positron (e⁺)
Same Mass but opposite Charge
Positron curls the other way from electron in a B Field

Accelerating Electrons Thru RF Cavities
A Circular Accelerator: Using B Field to Confine the electron and RF cavity to power it

Circular Track for Accelerating Electron

Accelerated electron through an effective voltage of 100 Billion Volts!
To be upgraded to 7 trillion Volts by 2007

Geneva Airport

Swiss Border

French Border

Geneva Airport
In Tunnel 150m underground, 27km ring of Magnets Keep electron in Circular Orbit

Inside A Circular Particle Accelerator Tunnel: Monorail!
Test of Relativistic Momentum In Circular Accelerator

\[ \vec{p} = \frac{m\vec{u}}{\sqrt{1-(u/c)^2}} = \gamma m\vec{u} \]

\[ \gamma \mu u = qBr = p \]

\[ \gamma = \frac{qBr}{\mu u} \]

Proof That \( p \neq \mu u \)

\[ \frac{1}{\sqrt{1-u^2/c^2}} \]

Relativistic Work Done & Change in Energy

\[ W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x} = \int_{x_1}^{x_2} \frac{d\vec{p}}{dt} \cdot d\vec{x} \]

\[ p = \frac{m\mu u}{\sqrt{1-u^2/c^2}} \]

\[ \frac{d\vec{p}}{dt} = \frac{m}{\sqrt{1-u^2/c^2}} \frac{du}{dt} \]

\[ \therefore W = \int_{u=0}^{u=1} m \frac{du}{dt} u dt \] (change in var x → u)

\[ W = \int_{0}^{u} \frac{m u du}{\left[1 - \frac{u^2}{c^2}\right]^{3/2}} = \frac{mc^2}{\gamma - 1} - mc^2 = \gamma mc^2 - mc^2 \]

Work done is change in kinetic energy \( K \)

\[ K = \gamma mc^2 - mc^2 \]

Total Energy \( E = \gamma mc^2 = K + mc^2 \)
But Professor… Why Can’s ANYTHING go faster than light?

\[ K = \left( \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} \right)^{1/2} - mc^2 \Rightarrow \left( K + mc^2 \right)^2 = \left( \frac{mc^2}{\left(1 - \frac{u^2}{c^2}\right)^{1/2}} \right)^2 \]

\[ \Rightarrow \left[1 - \frac{u^2}{c^2}\right] = m^2 c^4 \left[ K + mc^2 \right]^{-2} \]

\[ \Rightarrow u = c \sqrt{1 - \left( \frac{K}{mc^2} + 1 \right)^2} \text{ (Parabolic in } u \text{ Vs } \frac{K}{mc^2}) \]

As \( K \to \infty \), \( u \to c \)

Non-relativistic case: \( K = \frac{1}{2} mu^2 \Rightarrow u = \sqrt{\frac{2K}{m}} \)

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Relativistic Kinetic Energy Vs Velocity

\[ E_k = \frac{mc^2}{\sqrt{1 - \left(\frac{u}{c}\right)^2} - 1} \]

\[ E_k = \frac{1}{2} mu^2 \]

\[ \left( \frac{u}{c} \right)^2 \]
A Digression: How to Handle Large/Small Numbers

- Example: consider very energetic particle with very large Energy $E$

$$\gamma = \frac{E}{mc^2} = \frac{mc^2 + K}{mc^2} = 1 + \frac{K}{mc^2}$$

- Let’s Say $\gamma = 3 \times 10^{11}$, Now calculate $u$ from

$$\text{Try this on your el-cheapo calculator, you will get } \frac{u}{c} = 1, \ u = c \text{ due to limited precision.}$$

- In fact $u \cong c$ but not exactly!, try to get this analytically

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{1}{\sqrt{(1 - \beta)(1 + \beta)}}$$

Since $\beta = \frac{u}{c} \cong 1, \ 1 + \beta = 2$

$$\gamma = \frac{1}{\sqrt{2\sqrt{1 - \beta}}}$$

$$\Rightarrow 1 - \beta = \frac{1}{2\gamma^2} = 5 \times 10^{-24}, \ u = \beta c$$

$$\Rightarrow u = 0.9999999999999999995c!$$

Such particles are routinely produced in violent cosmic collisions

When Electron Goes Fast it Gets “Fat”

Total Energy $E = \gamma mc^2 = K + mc^2$

As $\frac{v}{c} \rightarrow 1, \ \gamma \rightarrow \infty$

Apparent Mass approaches $\infty$

New Concept
Rest Mass = particle mass when its at rest
Relativistic Kinetic Energy & Newtonian Physics

Relativistic KE \[ K = \gamma mc^2 - mc^2 \]

Remember Binomial Theorem

\[
\text{for } x << 1; \quad (1+x)^n = (1+ \frac{nx}{1!} + \frac{n(n-1)}{2!} x^2 + \text{smaller terms})
\]

\[
\therefore \quad \text{When } u << c, \quad \left[ 1 - \frac{u^2}{c^2} \right]^{-\frac{1}{2}} \approx 1 + \frac{1}{2} \frac{u^2}{c^2} + \text{...smaller terms}
\]

so \[ K \approx mc^2[1 + \frac{1}{2} \frac{u^2}{c^2}] - mc^2 = \frac{1}{2} \frac{mu^2}{c^2} \] (classical form recovered)

Total Energy of a Particle \[ E = \gamma mc^2 = KE + mc^2 \]

For a particle at rest, \( u = 0 \) \Rightarrow Total Energy \( E = mc^2 \)

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E=mc²: Sunshine Won’t Be Forever

Q: Solar Energy reaches earth at rate of 1.4kW per square meter of surface perpendicular to the direction of the sun. by how much does the mass of sun decrease per second owing to energy loss? The mean radius of the Earth’s orbit is 1.5 x 10¹¹m.

Surface area of a sphere of radius \( r \) is \( A = 4\pi r^2 \)

Total Power radiated by Sun = power received by a sphere whose radius is equal to earth’s orbit radius

\[
P = \frac{P}{A} = \frac{4\pi r^2}{4\pi r^2} = (1.4 \times 10^3 W / m^2)(4\pi)(1.5 \times 10^{11})^2
\]

\[ P = 4.0 \times 10^{26} W \]

So Sun loses \( E = 4.0 \times 10^{26} J \) of rest energy per second

Its mass decreases by \( \frac{E}{c^2} = \frac{4.0 \times 10^{26} J}{(3.0 \times 10^8)^2} = 4.4 \times 10^9 \text{ kg} \)

If the Sun's Mass = 2.0 \times 10^{30} kg So how long with the Sun last?
\[ E = \gamma mc^2 \Rightarrow E^2 = \gamma^2 m^2 c^4 \]

\[ p = \gamma mu \Rightarrow p^2 c^2 = \gamma^2 m^2 u^2 c^2 \]

\[ \Rightarrow E^2 - p^2 c^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^2 (c^2 - u^2) = \frac{m^2 c^2}{1 - \frac{u^2}{c^2}} (c^2 - u^2) = m^2 c^4 \]

\[ E^2 = p^2 c^2 + (mc^2)^2 \]

......important relation

For particles with zero rest mass like photon (EM waves)

\[ E = pc \text{ or } p = \frac{E}{c} \]

(light has momentum!)

Relativistic Invariance: \[ E^2 - p^2 c^2 = m^2 c^4 \]

: In all Ref Frames

Rest Mass is a "finger print" of the particle

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Mass Can “Morph” into Energy & Vice Verca

- Unlike in Newtonian mechanics
- In relativistic physics: Mass and Energy are the same thing
- New word/concept: MassEnergy, just like SpaceTime
- It is the mass-energy that is always conserved in every reaction: Before & After a reaction has happened
- Like squeezing a balloon: Squeeze here, it grows elsewhere
  - If you “squeeze” mass, it becomes (kinetic) energy & vice verca!
    - CONVERSION FACTOR = \( C^2 \)
    - This exchange rate never changes!