“Fixing” (upgrading) Newtonian mechanics

- To confirm with fast velocities
- Re-examine
  - Spacetime: $X, Y, Z, t$
  - Velocity: $V_x, V_y, V_z$
  - Momentum: $P_x, P_y, P_z$
  - (Proper) Rest Mass
  - Acceleration: $a_x, a_y, a_z$
  - Force
  - Work Done & Energy Change
  - Kinetic Energy
  - Mass IS energy

- Nuclear Fission
- Nuclear Fusion
- Making baby universes Learning about the first three minutes since the beginning of the universe
Lorentz Transformation Between Ref Frames

Lorentz Transformation

\[ x' = \gamma (x - vt) \]
\[ y' = y \]
\[ z' = z \]
\[ t' = \gamma \left( t - \frac{vx}{c^2} \right) \]

Inverse Lorentz Transformation

\[ x = \gamma (x' + vt) \]
\[ y = y' \]
\[ z = z' \]
\[ t = \gamma \left( t' + \frac{vx'}{c^2} \right) \]

As \( v \to 0 \), Galilean Transformation is recovered, as per requirement

Notice: SPACE and TIME Coordinates mixed up !!!
Lorentz Transform for Pair of Events

\[
\Delta x' = \gamma (\Delta x - v \Delta t) \\
\Delta t' = \gamma \left( \Delta t - \frac{v}{c^2} \Delta x \right) \\
\Delta x = \gamma (\Delta x' + v \Delta t') \\
\Delta t = \gamma \left( \Delta t' + \frac{v}{c^2} \Delta x' \right)
\]

S \rightarrow S' \quad S' \rightarrow S

Can understand Simultaneity, Length contraction & Time dilation formulae from this

Time dilation: Bulb in S frame turned on at \( t_1 \) & off at \( t_2 \) : What \( \Delta t' \) did S’ measure ?

two events occur at same place in S frame => \( \Delta x = 0 \)

\[
\Delta t' = \gamma \Delta t \quad (\Delta t = \text{proper time})
\]

Length Contraction: Ruler measured in S between \( x_1 \) & \( x_2 \) : What \( \Delta x' \) did S’ measure ?

two ends measured at same time in S' frame => \( \Delta t' = 0 \)

\[
\Delta x = \gamma (\Delta x' + 0) \Rightarrow \Delta x' = \Delta x / \gamma \quad (\Delta x = \text{proper length})
\]
Lorentz Velocity Transformation Rule

S and S’ are measuring ant’s speed u along x, y, z axes

In S' frame, \( u'_{x} = \frac{x'_{2} - x'_{1}}{t'_{2} - t'_{1}} = \frac{dx'}{dt'} \)

\[ dx' = \gamma(dx - vdt), \quad dt' = \gamma(dt - \frac{v}{c^{2}} dx) \]

\[ u'_{x} = \frac{dx - vdt}{dt - \frac{v}{c^{2}} dx}, \quad \text{divide by } dt' \]

\[ u'_{x} = \frac{u_{x} - v}{1 - \frac{vu_{x}}{c^{2}}} \]

For \( v << c \), \( u'_{x} = u_{x} - v \)

(Galilean Trans. Restored)
Does Lorentz Transform “work”? 

Two rockets travel in opposite directions.

An observer on earth (S) measures speeds = 0.75c and 0.85c for A & B respectively.

What does A measure as B’s speed?

Place an imaginary S’ frame on Rocket A ⇒ v = 0.75c relative to Earth Observer S.

\[ u'_x = \frac{u_x - v}{\sqrt{1 - \frac{u_x v}{c^2}}} = \frac{-0.850c - 0.750c}{\sqrt{1 - (-0.850c)(0.750c)}} = -0.977c \]

Consistent with Special Theory of Relativity.
Velocity Transformation Perpendicular to S-S’ motion

\[ dy' = dy, \quad dt' = \gamma\left(dt - \frac{v}{c^2} dx\right) \]

\[ u'_y = \frac{dy'}{dy'} = \frac{dy}{\gamma\left(dt - \frac{v}{c^2} dx\right)} \]

divide by dt on RHS

\[ u'_y = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2} u_x\right)} \]

Similarly

Z component of Ant' s velocity transforms as

\[ u'_z = \frac{u_z}{\gamma\left(1 - \frac{v}{c^2} u_x\right)} \]

There is a change in velocity in the direction \( \perp \) to S-S' motion!
Inverse Lorentz Velocity Transformation

Inverse Velocity Transform:

\[ u_x = \frac{u_{x'} + v}{1 + \frac{vu_{x'}}{c^2}} \]

\[ u_y = \frac{u_y'}{\gamma(1 + \frac{v}{c^2} u_x')} \]

\[ u_z = \frac{u_z'}{\gamma(1 + \frac{v}{c^2} u_x')} \]

As usual, replace \( V \) \( \Rightarrow \) \(-V\)
Example of Inverse velocity Transform

Biker moves with speed $= 0.8c$ past stationary observer

Throws a ball forward with speed $= 0.7c$

What does stationary observer see as velocity of ball?

Place $S'$ frame on biker

Biker sees ball speed

$u_{x'} = 0.7c$

Speed of ball relative to stationary observer

$u_x$ ?

$u_x = \frac{u_x' + v}{1 + \frac{u_x'v}{c^2}} = \frac{0.70c + 0.80c}{1 + \frac{(0.70c)(0.80c)}{c^2}} = 0.96c$
Can you be **seen** to be born before your mother?

A frame of Ref where sequence of events is REVERSED ?!!

$$\Delta t' = t'_2 - t'_1 = \gamma \left[ \Delta t - \left( \frac{v}{c^2} \frac{\Delta x}{c^2} \right) \right]$$

For what value of $v$ can $\Delta t' < 0$?
I Cant ‘be seen to arrive in SF before I take off from SD

\[ \Delta t' = t_2' - t_1' = \gamma \left[ \Delta t - \left( \frac{v \Delta x}{c^2} \right) \right] \]

For what value of \( v \) can \( \Delta t' < 0 \)

\[ \Delta t' < 0 \Rightarrow \Delta t < \frac{v \Delta x}{c^2} \Rightarrow 1 < \frac{v \Delta x}{c^2 \Delta t} = \frac{v u}{c^2} \]

\[ \Rightarrow \frac{v}{c} > \frac{c}{u} \Rightarrow v > c : \text{Not allowed} \]
Relativistic Momentum and Revised Newton’s Laws

Need to generalize the laws of Mechanics & Newton to confirm to Lorentz Transform and the Special theory of relativity: Example: \( \vec{p} = m\vec{u} \)

Before

\[ P = mv - mv = 0 \]

After

\[ P = 0 \]

\[ \begin{align*}
    v_1' &= \frac{v_1 - v}{1 - \frac{v_1v}{c^2}} = 0, \\
    v_2' &= \frac{v_2 - v}{1 - \frac{v_1v}{c^2}} = \frac{-2v}{1 + \frac{v^2}{c^2}}, \\
    V' &= \frac{V - v}{1 - \frac{Vv}{c^2}} = -v
\end{align*} \]

\[ \begin{align*}
    p_1' &= mv_1' + mv_2' = \frac{-2mv}{1 + \frac{v^2}{c^2}}, \\
    p_2' &= 2mV' = -2mv
\end{align*} \]

\[ p_1' \neq p_2' \]

Watching an Inelastic Collision between two putty balls.

\[ v_1' = 0 \]

\[ v_2' \]
Definition (without proof) of Relativistic Momentum

\[ \vec{p} = \frac{m\vec{u}}{\sqrt{1 - (u/c)^2}} = \gamma m\vec{u} \]

With the new definition relativistic momentum is conserved in all frames of references: Do the exercise.

New Concepts

Rest mass = mass of object measured in a frame of ref. where object is at rest

\[ \gamma = \frac{1}{\sqrt{1 - (u/c)^2}} \]

\(u\) is velocity of the object

NOT of a reference frame!