Time Dilation Example: Relativistic Doppler Shift

- Light: velocity $c = f \lambda$, $f = 1/T$
- A source of light $S$ at rest
- Observer $S'$ approaches $S$ with velocity $v$
- $S'$ measures $f'$ or $\lambda'$, $c = f' \lambda'$
- Expect $f' > f$ since more wave crests are being crossed by Observer $S'$ due to its approach direction than if it were at rest w.r.t source $S$
Relativistic Doppler Shift

Examine two successive wavefronts emitted by S at location 1 and 2.

In S’ frame, \( T' = \) time between two wavefronts.

In time \( T' \), the Source moves by \( cT' \) w.r.t 1.

Meanwhile Light Source moves a distance \( vT' \).

Distance between successive wavefront

\( \lambda' = cT' - vT' \)

\[ \lambda' = cT' - vT', \quad \text{use } f = c/\lambda \]

\[ f' = \frac{c}{(c-v)T'}, \quad T' = \frac{T}{\sqrt{1-(v/c)^2}} \]

Substituting for \( T' \), use \( f = 1/T \)

\[ \Rightarrow f' = \frac{\sqrt{1-(v/c)^2}}{1-(v/c)} \]

\[ \Rightarrow f' = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} \quad f \]

better remembered as:

\[ f_{\text{obs}} = \frac{\sqrt{1+(v/c)}}{\sqrt{1-(v/c)}} \quad f_{\text{source}} \]

\[ f_{\text{obs}} = \text{Freq measured by observer approaching light source} \]
\[ f_{\text{obs}} = \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} \frac{f_{\text{source}}}{f_{\text{source}}} \]
Emission & Absorption Spectra of Atoms: DNA

The diagram illustrates the energy levels of an atom, focusing on the transition between different energy states. The energy levels are marked by different quantum numbers ($n$), with transitions occurring between $n = 1$, $n = 2$, $n = 3$, and $n = 4$. The transitions are categorized as absorption, emission, and ionization. The energy differences are indicated at the right side of the diagram, with specific labels for ultraviolet, visible, and infrared regions. The Lyman, Balmer, and Paschen series are also highlighted, with the energy differences shown as 13.6 eV, 12.8 eV, 12.1 eV, and 10.2 eV.
Spectral Lines and Tracking Moving Objects

Object at rest

Moving object

Observer A

Observer B

Frequency of signal

Spectrum of source

Listener

Red shift for A

Blue shift for B
Doppler Shift in Spectral Lines and Motion of Radiating Objects

Laboratory Spectrum, lines at rest wavelengths

Lines Redshifted, Object moving away from me

Larger Redshift, object moving away even faster

Lines blueshifted, Object moving towards me

Larger blueshift, object approaching me faster
The Cosmological Redshift
The Expanding Universe: Space itself is Expanding
Cosmological Redshift

As Universe expands EM waves stretch in Wavelength
Seeing Distant Galaxies Thru Hubble Telescope

Through center of a massive galaxy clusters Abell 1689
Galaxies at different locations in our Universe travel at different velocities.

<table>
<thead>
<tr>
<th>Galaxies</th>
<th>H + K</th>
<th>Velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virgo</td>
<td></td>
<td>1200 km/s</td>
</tr>
<tr>
<td>Ursa Major</td>
<td></td>
<td>15,000 km/s</td>
</tr>
<tr>
<td>Corona Borealis</td>
<td></td>
<td>22,000 km/s</td>
</tr>
<tr>
<td>Boötes</td>
<td></td>
<td>39,000 km/s</td>
</tr>
<tr>
<td>Hydra</td>
<td></td>
<td>61,000 km/s</td>
</tr>
</tbody>
</table>
Hubble’s Measurement of Recessional Velocity of Galaxies

\[ V = H \cdot d \]  
Farther things are faster they go

\[ H = 75 \, \text{km/s/Mpc} \]  
\[ (3.08 \times 10^{16} \, \text{m}) \]

Play the movie backwards!
Our Universe is about 10 Billion Years old
A paradox is an apparently self-contradictory statement, the underlying meaning of which is revealed only by careful scrutiny. The purpose of a paradox is to arrest attention and provoke fresh thought.

``A paradox is not a conflict within reality. It is a conflict between reality and your feeling of what reality should be like." – Richard Feynman

Construct a few paradoxes in Relativity & analyze them
Jack and Jill’s Excellent Adventure: Twin Paradox

Jack & Jill are 20 yr old twins, with same heartbeat

Jack takes off with $V = 0.8c$ to a star 20 light years away

Jill stays behind, watches Jack by telescope

Jill sees Jack’s heart slow down

Factor:  
$$\frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.8c/c)^2}} = 0.6$$

For every 5 beats of her heart
She sees Jack’s beat only 3!

Jack has only 3 thoughts for 5 that Jill has! Every things slows!

Finally Jack **returns** after 50 yrs gone by according to Jill’s calendar

Only 30 years have gone by Jack
Jack is 50 years old, Jane is 70!

Is there a paradox here??
Twin Paradox?

- Paradox: Turn argument around, motion is relative
- Jack claims he at rest, Jill is moving $v=0.8c$
- Should not Jill be 50 years old when 70 year old Jack returns from space Odyssey?
- No! …because Jack is not traveling in an inertial frame of reference
  - TO GET BACK TO EARTH HE HAS TO TURN AROUND => decelerate/accelerate
- But Jill always remained in Inertial frame
- Time dilation formula applies to Jill’s observation of Jack but not to Jack’s observation of Jill

Non-symmetric aging verified with atomic clocks taken on airplane trip around world and compared with identical clock left behind. Observer who departs from an inertial system will always find its clock slow compared with clocks that stayed in the system.
Round The World With An Atomic Clock

- Atomic Clock: certain transitions in Cesium atom
- Two planes take off from DC, travel east and west
  - Eastward trip took 41.2 hrs
  - Westward trip took 48.6 hrs
- Atomic clocks compared to similar ones kept in DC
- Need to account for Earth’s rotation + GR etc

<table>
<thead>
<tr>
<th>Travel</th>
<th>Predicted</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eastward</td>
<td>-40 ± 23 ns</td>
<td>-59 ± 10 ns</td>
</tr>
<tr>
<td>Westward</td>
<td>275 ± 21 ns</td>
<td>273 ± 7 ns</td>
</tr>
</tbody>
</table>

Flying clock ticked faster or slower than reference clock. Slow or fast is due to Earth’s rotation.
Fitting a 5m pole in a 4m barnhouse

Student with pole runs with $v = \frac{3}{5}c$

- Farmboy sees pole contraction factor
  \[ \sqrt{1 - \left(\frac{3c}{5c}\right)^2} = \frac{4}{5} \]
  says pole just fits in the barn fully!

- Student sees barn contraction factor
  \[ \sqrt{1 - \left(\frac{3c}{5c}\right)^2} = \frac{4}{5} \]
  says barn is only 3.2m long, too short to contain entire 5m pole!

Farmboy says “You can do it”
Student says “Dude, you are nuts”

Is there a contradiction? Is Relativity wrong?

Homework: You figure out who is right, if any and why.

Hint: Think in terms of observing two events
  Arrival of left end of pole at left end of barn
  Arrival of right end of pole at right end of barn
Discovering The Correct Transformation Rule

\[ x' = x - vt \quad \text{guess} \rightarrow \quad x' = G(x - vt) \]
\[ x = x' + vt' \quad \text{guess} \rightarrow \quad x = G(x' + vt') \]

Need to figure out functional form of \( G \)!

G must be dimensionless
G does not depend on \( x,y,z,t \)
But G depends on \( v/c \)
G is symmetric
As \( v/c \rightarrow 0 \), \( G \rightarrow 1 \)

Do a Thought Experiment: Rocket Motion along x axis

Rocket in \( S' (x',y',z',t') \) frame moving with velocity \( v \) w.r.t observer on frame \( S (x,y,z,t) \)
Flashbulb mounted on rocket emits pulse of light at the instant origins of \( S,S' \) coincide
That instant corresponds to \( t = t' = 0 \). Light travels as a spherical wave, origin is at \( O,O' \)

Speed of light is \( c \) for both observers

Examine a point \( P \) (at distance \( r \) from \( O \) and \( r' \) from \( O' \)) on the Spherical Wavefront
The distance to point \( P \) from \( O : r = ct \)
The distance to point \( P \) from \( O' : r' = ct' \)

Clearly \( t \) and \( t' \) must be different
\[ t \neq t' \]
Discovering Lorentz Transformation for \((x,y,z,t)\)

Motion is along \(x-x'\) axis, so \(y\), \(z\) unchanged

\[ y' = y, \quad z' = z \]

Examine points \(x\) or \(x'\) where spherical wave crosses the horizontal axes: \(x = r\), \(x' = r'\)

\[
\begin{align*}
x &= ct = G(x' + vt') \\
x' &= ct' = G(x - vt), \\
\Rightarrow t' &= \frac{G}{c} (x - vt) \\
\therefore x &= ct = G(ct' + vt') \\
\therefore ct &= G^2 \left[(ct - vt) + vt - \frac{v^2}{c} t\right] \\
\Rightarrow c^2 &= G^2 [c^2 - v^2] \\
\therefore G &= \frac{1}{\sqrt{1 - (v/c)^2}} = \gamma \\
\therefore x' &= \gamma(x - vt) \\
\end{align*}
\]

\[
\begin{align*}
x' &= \gamma(x - vt), \quad x = \gamma(x' + vt') \\
\Rightarrow x &= \gamma(\gamma(x - vt) + vt') \\
\therefore x - \gamma^2 x + \gamma^2 vt &= \gamma vt' \\
\therefore t' &= \frac{x}{\gamma v} - \frac{\gamma^2 x + \gamma^2 vt}{\gamma v} = \gamma \left[\frac{x}{\gamma^2 v} - \frac{x}{v} + t\right] \\
\therefore t' &= \gamma \left[t + \frac{x}{v} \left(\frac{1}{\gamma^2 - 1}\right)\right], \text{ since } \left(\frac{1}{\gamma^2 - 1}\right) = -\left(\frac{v}{c}\right)^2 \\
\Rightarrow t' &= \gamma \left[t + \frac{x}{v} \left[1 - \left(\frac{v}{c}\right)^2\right] - 1\right] = \gamma \left[t - \left(\frac{vx}{c^2}\right)\right]
\end{align*}
\]
Lorentz Transformation Between Ref Frames

Lorentz Transformation

\[
x' = \gamma (x - vt)
\]
\[
y' = y
\]
\[
z' = z
\]
\[
t' = \gamma \left( t - \frac{vx}{c^2} \right)
\]

Inverse Lorentz Transformation

\[
x = \gamma (x' + vt)
\]
\[
y = y'
\]
\[
z = z'
\]
\[
t = \gamma \left( t' + \frac{vx'}{c^2} \right)
\]

As \( v \to 0 \), Galilean Transformation is recovered, as per requirement

Notice: SPACE and TIME Coordinates mixed up !!!