Continuous & discrete spectra of Elements

- Hot blackbody
- Cloud of cooler gas
- Prism

a. Continuous spectrum
b. Absorption line spectrum
c. Emission line spectrum
Visible Spectrum of Sun Through a Prism
Emission & Absorption Line Spectra of Elements

Source of wavelengths $\lambda_1$ and $\lambda_2$ ($\lambda_2 > \lambda_1$)

(a)

Hydrogen

H$_\alpha$, H$_\beta$, H$_\gamma$, H$_\delta$, H$_\epsilon$, f, H$_\infty$
D lines darken noticeably when Sodium vapor introduced between slit and prism.
Emission & Absorption Line Spectrum of Elements

- Emission line appear dark because of photographic exposure

Absorption spectrum of Na
While light passed thru Na vapor is absorbed at specific $\lambda$
Spectral Observations: series of lines with a pattern

- Empirical observation (by trial & error)
- All these series can be summarized in a simple formula

\[ \frac{1}{\lambda} = R \left( \frac{1}{n_f^2 - n_i^2} \right), n_f > n_i, n_i = 1, 2, 3, 4 \ldots \]

Fitting to spectral line series data

\[ R = 1.09737 \times 10^7 \, m^{-1} \]

How does one explain this?
Rutherford Atom & Classical Physics
Atom: The Classical disaster

Radiated light of ever shorter $\lambda$
1. Electron in circular orbit around proton with vel=\(v\)
2. Only stationary orbits allowed. Electron does not radiate when in these stable (stationary) orbits
3. Orbits quantized:
\[ M_e v r = n \frac{h}{2\pi} \ (n=1,2,3\ldots) \]
4. Radiation emitted when electron “jumps” from a stable orbit of higher energy \(\rightarrow\) stable orbit of lower energy \(E_f - E_i = hf = \frac{hc}{\lambda}\)
5. Energy change quantized
\[ f = \text{frequency of radiation} \]
Reduced Mass of 2-body system

- Both Nucleus & $e^-$ revolve around their common center of mass (CM).
- Such a system is equivalent to a single particle of “reduced mass” $\mu$ that revolves around the position of Nucleus at a distance of ($e^- - N$) separation.

$$\mu = \frac{m_e M}{m_e + M}, \text{ when } M \gg m, \mu = m \text{ (Hydrogen atom)}$$

Not so when calculating Muon ($m_\mu = 207 \ m_e$) or equal mass charges rotating around each other (similar to what you saw in gravitation).
E=KE+U = \frac{1}{2} m_e v^2 - k \frac{e^2}{r}

Force Equality for Stable Orbit

⇒ Coulomb attraction = CP Force

\[ k \frac{e^2}{r^2} = \frac{m_e v^2}{r} \]

⇒ KE = \frac{m_e v^2}{2} = k \frac{e^2}{2r}

Total Energy E = KE+U = -k \frac{e^2}{2r}

Negative E ⇒ Bound system

This much energy must be added to the system to break up the bound atom

Radius of Electron Orbit:

\[ mvr = n\hbar \]

⇒ \[ v = \frac{n\hbar}{mr} \]

substitute in KE= \frac{1}{2} m_e v^2 = \frac{k e^2}{2r}

⇒ \[ r_n = \frac{n^2 \hbar^2}{m k e^2} \], \( n \) = 1, 2, ..., \( \infty \)

n = 1 ⇒ Bohr Radius \( a_0 \)

\[ a_0 = \frac{1^2 \hbar^2}{m k e^2} = 0.529 \times 10^{-10} \text{ m} \]

In general \( r_n = n^2 a_0 \); \( n \) = 1, 2, ..., \( \infty \)

Quantized orbits of rotation
Energy Level Diagram and Atomic Transitions

\[ E_n = K + U = \frac{-ke^2}{2r} \]

since \( r_n = a_0n^2, n = \text{quantum number} \)

\[ E_n = \frac{-ke^2}{2a_0n^2} = -\frac{13.6}{n^2} \text{eV}, \quad n = 1, 2, 3, \ldots \infty \]

Interstate transition: \( n_i \rightarrow n_f \)

\[ \Delta E = hf = E_i - E_f \]

\[ \Delta E = -\frac{ke^2}{2a_0} \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \]

\[ f = \frac{ke^2}{2hca_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2hca_0} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

\[ \lambda = \frac{R}{\left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)} \]

Diagram showing energy levels and transitions.
Hydrogen Spectrum: as explained by Bohr

\[ E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2} \]

Bohr’s “R”
Same as the Rydberg Constant
derived empirically from
Spectral series
Another Look at the Energy levels

\[ E_n = -\left(\frac{ke^2}{2a_0}\right)\frac{Z^2}{n^2} \]
Bohr’s Atom: Emission & Absorption Spectra

a  Absorption

b  Emission

$n = 2$

$n = 3$
Some Notes About Bohr Like Atoms

- Ground state of Hydrogen atom \( n=1 \) \( E_0 = -13.6 \text{ eV} \)
- Method for calculating energy levels etc applies to all Hydrogen-like atoms \( \rightarrow -1e \text{ around } +Ze \)
  - Examples: He\(^+\), Li\(^{++}\)
- Energy levels would be different if replace electron with Muons
- Bohr’s method can be applied in general to all systems under a central force (e.g. gravitational instead of Coulombic)
  \[ U(r) = k \frac{Q_1 Q_2}{r} \rightarrow G \frac{M_1 M_2}{r} \]
  Changes every thing: \( E \), \( r \), \( f \) etc
  "Importance of constants in your life"
Atomic Excitation by Electrons: Franck-Hertz Expt