See Chapters 1 & 2 of Feynman Lectures in Physics Vol III
Or Six Easy Pieces by Richard Feynman : Addison Wesley Publishers
An Experiment with Indestructible Bullets

Probability $P_{12}$ when both holes open

Erratic Machine gun sprays in many directions

Made of Armor plate

$P_{12} = P_1 + P_2$
An Experiment With Water Waves

Measure Intensity of Waves
(by measuring amplitude of displacement)

Intensity $I_{12}$ when Both holes open

$I_{12} = |h_1 + h_2|^2 = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \delta$
Interference Phenomenon in Waves

\[ n\lambda = d \sin \theta \]
An Experiment With Electrons

Probability $P_{12}$ when Both holes open

$P_{12} \neq P_1 + P_2$
Interference in Electrons Thru 2 slits

Growth of 2-slit Interference pattern thru different exposure periods
Photographic plate (screen) struck by:

28 electrons

1000 electrons

10,000 electrons

10^6 electrons

White dots simulate presence of electron
No white dots at the place of destructive Interference (minima)
Probability $P_{12}$ when both holes open and I see which hole the electron came thru.

$$P'_{12} = P'_1 + P'_2$$
Probability $P_{12}$ when both holes open and I see which hole the electron came thru.
Watching The Electrons With Dim Light

Probability $P_{12}$ when both holes open and I don’t see which hole the electron came thru.
Compton Scattering: Shining light to observe electron

\[ \lambda = \frac{h}{p} = \frac{hc}{E} = \frac{c}{f} \]

The act of Observation DISTURBS the object being watched, here the electron moves away from where it was originally
Watching Electrons With Light of $\lambda >>$ slitsize but High Intensity

Probability $P_{12}$ when both holes open but cant tell from flash which hole the electron came thru
Why Fuzy Flash? → Resolving Power of Light

Image of 2 separate point sources formed by a converging lens of diameter $d$, ability to resolve them depends on $\lambda$ & $d$ because of the Inherent diffraction in image formation.

Resolving power $\Delta x \approx \frac{\lambda}{2\sin \theta}$
1. Probability of an event is given by the square of amplitude of a complex # Ψ: Probability Amplitude

2. When an event occurs in several alternate ways, probability amplitude for the event is sum of probability amplitudes for each way considered separately. There is interference:

\[ \Psi = \Psi_1 + \Psi_2 \]

\[ P_{12} = |\Psi_1 + \Psi_2|^2 \]

3. If an experiment is done which is capable of determining whether one or other alternative is actually taken, probability for event is just sum of each alternative

- Interference pattern is LOST!
Is There No Way to Beat Uncertainty Principle?

• How about NOT watching the electrons!
• Lets be a bit crafty
• Since this is a Thought experiment → ideal conditions
  – Mount the wall on rollers, put a lot of grease → frictionless
  – Wall will move when electron hits it
  – Watch recoil of the wall containing the slits when the electron hits it
  – By watching whether wall moved up or down I can tell
    • Electron went thru hole # 1
    • Electron went thru hole #2

• Will my ingenious plot succeed?
Measuring The Recoil of The Wall: Not Watching Electron!
Losing Out To Uncertainty Principle

- To measure the RECOIL of the wall ⇒
  - must know the initial momentum of the wall before electron hit it
  - Final momentum after electron hits the wall
  - Calculate vector sum → recoil

- Uncertainty principle:
  - To do this ⇒ \( \Delta P = 0 \) → \( \Delta X = \infty \) [can not know the position of wall exactly]
  - If don’t know the wall location, then down know where the holes are
  - Holes will be in different place for every electron that goes thru
  - The center of interference pattern will have different (random) location for each electron
  - Such random shift is just enough to Smear out the pattern so that no interference is observed!

- Uncertainty Principle Protects Quantum Mechanics!
The Bullet Vs The Electron: Each Behaves the Same Way
Quantum Mechanics of Subatomic Particles

- Act of Observation destroys the system (No watching!)
- If can’t watch then All conversations can only be in terms of Probability $P$
- Every particle under the influence of a force is described by a Complex wave function $\Psi(x,y,z,t)$
- $\Psi$ is the ultimate DNA of particle: contains all info about the particle under the force (in a potential e.g Hydrogen)
- Probability of per unit volume of finding the particle at some point $(x,y,z)$ and time $t$ is given by
  - $P(x,y,z,t) = \Psi(x,y,z,t) \cdot \Psi^*(x,y,z,t) = |\Psi(x,y,z,t)|^2$
- When there are more than one path to reach a final location then the probability of the event is
  - $\Psi = \Psi_1 + \Psi_2$
  - $P = |\Psi^* \Psi| = |\Psi_1|^2 + |\Psi_2|^2 + 2|\Psi_1|\Psi_2| \cos \phi$
Wave Function of “Stuff” & Probability Density

- Although not possible to specify with certainty the location of particle, it's possible to assign probability $P(x)dx$ of finding particle between $x$ and $x+dx$
- $P(x) \, dx = |\Psi(x,t)|^2 \, dx$
- E.g. intensity distribution in light diffraction pattern is a measure of the probability that a photon will strike a given point within the pattern

Probability of a particle to be in an interval $a \leq x \leq b$ is area under the curve from $x=a$ to $x=b$
Ψ: The Wave function Of A Particle

• The particle must be some where
  \[
  \int_{-\infty}^{+\infty} |\psi(x, t)|^2 \, dx = 1
  \]
• Any \( \Psi \) satisfying this condition is NORMALIZED
• Prob of finding particle in finite interval
  \[
  P(a \leq x \leq b) = \int_{a}^{b} \psi^* (x, t) \psi(x, t)\, dx
  \]
• Fundamental aim of Quantum Mechanics
  – Given the wavefunction at some instant (say \( t=0 \)) find \( \Psi \) at some subsequent time \( t \)
  – \( \Psi(x,t=0) \rightarrow \Psi(x,t) \) …evolution
  – Think of a probabilistic view of particle's “newtonian trajectory”
    • We are replacing Newton’s 2\(^{nd}\) law for subatomic systems

The Wave Function is a mathematical function that describes a physical object \( \rightarrow \) Wave function must have some rigorous properties:

• \( \Psi \) must be finite
• \( \Psi \) must be continuous fn of \( x,t \)
• \( \Psi \) must be single-valued
• \( \Psi \) must be smooth fn \( \frac{d\psi}{dx} \) must be continuous

WHY?
Bad (Mathematical) Wave Functions: You Decide Why
A Simple Wave Function : Free Particle

- Imagine a free particle of mass $m$, $p$ and $K=p^2/2m$
- Under no force, no attractive or repulsive potential to influence it
- Particle does what it pleases: can be anywhere $[-\infty \leq x \leq +\infty ]$
  - No relationship, no mortgage, no quiz, no final exam..it's essentially a bum!
  - How to describe a quantum mechanical bum?

- $\Psi(x,t)= Ae^{i(kx-\omega t)} = A(Cos(kx-\omega t)+isin(kx-\omega t))$

$$k = \frac{p}{\hbar}; \quad \omega = \frac{E}{\hbar}$$

For non-relativistic particles

Has definite momentum and energy but location unknown!

$$E=\frac{p^2}{2m} \Rightarrow \omega(k)=\frac{\hbar k^2}{2m}$$
Wave Function of Free Particle: Wave Packet

Sum of Plane Waves:

\[ \Psi(x, 0) = \int_{-\infty}^{+\infty} a(k) e^{i k x} \, dk \]

\[ \Psi(x, t) = \int_{-\infty}^{+\infty} a(k) e^{i(kx - \omega t)} \, dk \]

Wave Packet initially localized in \( \Delta X, \Delta t \) undergoes dispersion.
Where Do Wave Functions Come From

- Are solutions of the time dependent Schrodinger Equation
- Given a potential $U(x) \rightarrow$ particle under certain force

\[
\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + U(x)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}
\]