Speed of Light, $c = 3.0 \times 10^8 \text{m/s}$
Planck's Constant, $\hbar = 6.626 \times 10^{-34} \text{J} \times \text{S}$
Planck's Constant, $\hbar = 4.136 \times 10^{-15} \text{eV} \times \text{s}$

Ground state energy $E_1$ for particle in a box $= \frac{\hbar^2}{2mL^2}$

Wavefunction for particle in a box $= \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right)$

Atomic Mass Unit $u = 1.6606 \times 10^{-27} \text{Kg}$ or 931.5 MeV/c$^2$
Proton Rest Mass $= 1.673 \times 10^{-27} \text{Kg}$ or 1.0073 u
Neutron Rest Mass $= 1.675 \times 10^{-27} \text{Kg}$ or 1.009 u

\[
\int \sin x \, dx = -\cos x
\]
\[
\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4}
\]
\[
\int x \sin^2 x \, dx = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}
\]
\[
\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4}
\]
\[
\int x \cos^2 x \, dx = \frac{x^2}{2} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8}
\]

Pl. consult the proctor if you have difficulty understanding the question.
Problem [1]. Jimmy Neutron: Trapped Genius (10 points):

The nuclear potential that binds protons and neutrons in the nucleus of an atom is often approximated by a square well. Imagine a neutron confined in an infinite square well of length $10^{-15}$ m, a typical nuclear diameter. (a) Draw an energy level diagram for the neutron for levels up to $n = 4$. (b) Find the wavelengths and (c) the energy associated with all photons that can be emitted by the neutron in making the transitions that would eventually get it from $n=4$ state to $n=1$ state. Indicate the transitions on the energy level diagram.

Problem [2]. Just Another Brick in The Wall (10 points):

A particle of mass $m$ moves in a two-dimensional box of length $L_1$ and width $L_2$. (a) Write the expression for the normalized wavefunctions and (b) energies as a function of quantum numbers $n_1$ and $n_2$. (Assume that the box is in x-y plane.) (c) Find the energies of the ground state and the first three excited states.
infinite square well

\[ L = 10^{-15} \text{m} \quad m = 1.009 u = 1.009 \times 931.5 \text{MeV} \frac{\text{MeV}}{c^2} \]

\[ E_n = \frac{\pi^2 k^2}{2 mL^2} - \frac{\pi^2 (4.136 \times 10^{-15} \text{eV} \cdot \text{s})^2 (3 \times 10^8 \text{m/s})^2}{2(1.009)(931.5 \text{MeV})(10^{-15} \text{m})^2} = 8.1 \times 10^9 \text{eV} \]

\[ E_n = (8.1 \times 10^9 \text{eV}) n^2 = (1.3 \times 10^{-9} \text{J}) n^2 \]

\[ \begin{array}{c}
  n=4 & E_4 \\
  n=3 & E_3 \\
  n=2 & E_2 \\
  n=1 & E_1 \\
\end{array} \]

\[ \begin{array}{c}
  T_1 \\
  T_2 \\
  T_3 \\
  T_4 \\
\end{array} \]

\[
\begin{align*}
  E_1 &= 8.1 \times 10^9 \\
  E_2 &= 3.2 \times 10^9 \\
  E_3 &= 7.3 \times 10^9 \\
  E_4 &= 1.3 \times 10^{10} \\
  J_1 &= 1.3 \times 10^{-9} \\
  J_2 &= 6.2 \times 10^{-9} \\
  J_3 &= 1.2 \times 10^{-8} \\
  J_4 &= 2.1 \times 10^{-8} \\
\end{align*}
\]

\(T_1, T_2, T_3, T_4\) indicate the 4 kinds of transitions we could have from \(n=4\) to \(n=1\) state.
from the diagram we see that 6 different photons can be emitted during transitions from \( n=4 \) to \( n=1 \):

- \( 4 \rightarrow 1 \)
- \( 4 \rightarrow 2 \)
- \( 4 \rightarrow 3 \)
- \( 3 \rightarrow 2 \)
- \( 3 \rightarrow 1 \)
- \( 2 \rightarrow 1 \)

The energies are given by \( E_i - E_f \),

the wavelengths are given by \( \frac{hc}{E_i - E_f} \)

<table>
<thead>
<tr>
<th>( n_i \rightarrow n_f )</th>
<th>Energy (eV)</th>
<th>Wavelength (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 4 \rightarrow 1 )</td>
<td>1.2 x 10^-10</td>
<td>1.0 x 10^-12</td>
</tr>
<tr>
<td>( 4 \rightarrow 2 )</td>
<td>9.8 x 10^-10</td>
<td>1.3 x 10^-12</td>
</tr>
<tr>
<td>( 4 \rightarrow 3 )</td>
<td>5.7 x 10^-10</td>
<td>2.2 x 10^-12</td>
</tr>
<tr>
<td>( 3 \rightarrow 2 )</td>
<td>4.1 x 10^-10</td>
<td>3.0 x 10^-12</td>
</tr>
<tr>
<td>( 3 \rightarrow 1 )</td>
<td>6.5 x 10^-10</td>
<td>1.9 x 10^-12</td>
</tr>
<tr>
<td>( 2 \rightarrow 1 )</td>
<td>2.4 x 10^-10</td>
<td>5.2 x 10^-12</td>
</tr>
</tbody>
</table>
\[ a \quad \psi(x, y) = \sqrt{\frac{2}{L_1}} \sqrt{\frac{2}{L_2}} \sin\left(\frac{n_1 \pi x}{L_1}\right) \sin\left(\frac{n_2 \pi y}{L_2}\right) \]

\[ b \quad E_{n_1, n_2} = \frac{\pi^2 k^2}{2m} \left[ \left(\frac{n_1}{L_1}\right)^2 + \left(\frac{n_2}{L_2}\right)^2 \right] \]

\[ c \quad \text{ground state} \\ n_1 = 1, n_2 = 1 \\ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left[ \left(\frac{1}{L_1}\right)^2 + \left(\frac{1}{L_2}\right)^2 \right] \]

\[ \text{if } L_1 > L_2 \text{ then} \]

1st excited state \\ \[ n_1 = 1, n_2 = 2 \] \\ \[ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left(\frac{1}{L_1^2} + \frac{4}{L_2^2}\right) \]

2nd \\ \[ n_1 = 2, n_2 = 1 \] \\ \[ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left(\frac{4}{L_1^2} + \frac{1}{L_2^2}\right) \]

3rd \\ \[ n_1 = 2, n_2 = 2 \] \\ \[ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left(\frac{4}{L_1^2} + \frac{4}{L_2^2}\right) \]

\[ \text{if } L_1 < L_2 \text{ then} \]

1st excited state \\ \[ n_1 = 2, n_2 = 1 \] \\ \[ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left(\frac{4}{L_1^2} + \frac{1}{L_2^2}\right) \]

2nd \\ \[ n_1 = 1, n_2 = 2 \] \\ \[ \mathcal{E} = \frac{\pi^2 k^2}{2m} \left(\frac{1}{L_1^2} + \frac{4}{L_2^2}\right) \]

3rd \\ \[ n_1 = 2, n_2 = 2 \] \\ \[ \text{see above for } \mathcal{E} \]