Some Useful Numbers, Equations and Identities

Planck's Constant, $\hbar = 6.626 \times 10^{-34} \text{ J} \times \text{ s}$

Planck's Constant, $\hbar = 4.136 \times 10^{-15} \text{ eV} \times \text{ s}$

$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

$\Delta x \Delta p \approx \hbar$

Gravitational Constant, $G = 6.67 \times 10^{-11} \text{ N} \times \text{ m}^2/\text{ kg}^2$

Gravitational Force $F = G \frac{M_1M_2}{r^2}$

Electron Mass $m_e = 9.11 \times 10^{-31} \text{ Kg}$

Electron Charge $e = 1.602 \times 10^{-19} \text{ C}$

Proton Rest Mass $m_p = 1.67 \times 10^{-27} \text{ Kg}$ or 1.0073 $u$

In the Hydrogen like atom:

Orbit radius $r_n = \frac{n^2 \hbar^2 c_0}{\pi m_e e^2}$

Bohr radius $a_0 = \frac{\hbar^2 c_0}{\pi m_e e^2} = 0.529 \text{ nm}$

Energy $E_n = \frac{-m_e e^4}{8 \varepsilon_0 \hbar^2 n^2}$ for $n = 1, 2, 3, 4, ...$

Binding Energy ($n = 1$) of the Hydrogen Atom $=-2.18 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$

Pl. consult the proctor if you have difficulty understanding the question.
Problem [1] **Tiger Hunting in a Quantum Jungle** (8 points): Somewhere in the Himalayan mountain range there are rumors of a mysterious Quantum jungle where the value of the Planck's constant $\hbar$ is much larger than our usual world. Suppose that you are in this quantum jungle where $\hbar = 50 \text{ J.s.}$ "Fuzzy" the tiger runs past you in the bushes a few meters away. The tiger, weighing 30kg, is known to be in a region about 4m long. (a) What is the minimum uncertainty in his speed? (b) Assuming this uncertainty in his speed to prevail for 10 seconds, determine the uncertainty in his position after this time.

Problem [2] **Basketball Team From Another Universe** (12 points):
You are the way you are because of a handful of fundamental physical constants. If they change, you change too! So figure this:

In an alien universe, matter like the electron, has no electric charge so the electric force between particles does not exist. In such a universe, the electron is bound to the proton, as in Bohr's Hydrogen atom, but by the gravitational force (like between the sun and the earth) rather than by the electric force. What would be (a) radius (b) energy of the first Bohr orbit of such a bound system. (c) Would people made of such atoms be taller than Michael Jordan? The size of our universe is about $10^{26}$m.
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\[ h = 50 \text{ J.s} \quad m = 30 \text{ kg} \]

(\text{a}) \quad \Delta x \Delta p \approx h \quad \Delta p = m \Delta v

\[ \Delta v = \frac{h}{m \Delta x} \quad \text{with} \quad \Delta x = 4 \text{ meters} \]

\[ = \frac{50 \text{ kg m}^2/\text{s}^2 \cdot \text{s}}{30 \text{ kg} \times 4 \text{ m}} = 0.42 \text{ m/s} \]

(\text{b}) \quad \Delta x' = \Delta v \cdot t

\[ t = 10 \text{ s} \]

\[ \Delta v = 0.42 \text{ m/s} \]

\[ \Delta x \rightarrow \Delta x + \Delta x' \Rightarrow \text{uncertainty in position} = 8.2 \text{ m} \]

2

\( F = \frac{G m e m p}{r^2} = \frac{m e v^2}{r} \Rightarrow v^2 = \frac{G m p}{r} \)

\( \lambda = \frac{h}{m e v} \quad \text{de Broglie} \)

\[ n \lambda = 2\pi r \quad \text{quantize wavelengths} \]

\( \Rightarrow 2\pi r = \frac{n h}{m e V} \)

\[ r = \frac{n h}{2\pi m e V} \]
\( r^2 = \left( \frac{n^2 \hbar}{(2\pi m_e)} \right)^2 \frac{1}{v^2} = \left( \frac{n^2 \hbar}{(2\pi m_e)} \right)^2 \frac{r}{G m_p} \)

\( r = \left( \frac{\hbar}{2\pi m_e} \right)^2 \frac{1}{G m_p} \)

first Bohr orbit \( \Rightarrow n = 1 \) then \( r = 1.2 \times 10^{-2} \text{ m} \)

\( E = \frac{1}{2} m_e v^2 - \frac{G m e m_p}{r} \)

\( = \frac{1}{2} m e m_p G \frac{1}{r} - \frac{G m e m_p}{r} \) use \( v^2 \) from (a)

\( = -\frac{1}{2} \frac{G m e m_p}{r} \) use \( r = 1.2 \times 10^{-2} \text{ m} \)

\( = -4.2 \times 10^{-77} \text{ J} = -2.6 \times 10^{-77} \text{ eV} \)

C. People would be taller