UCSD  DEPARTMENT OF PHYSICS

PHYSICS 2D, Winter 2002

PROF. SHARMA

Some Useful Numbers, Equations and Identities

Speed of Light, \( c = 3.0 \times 10^8 \text{m/s} \)

Planck’s Constant, \( h = 6.626 \times 10^{-34} \text{J} \times \text{s} \)

Planck’s Constant, \( \hbar = 4.136 \times 10^{-15} \text{eV} \times \text{s} \)

1 eV = \( 1.60 \times 10^{-19} \text{J} \)

Compton Wavelength for scattering off electron = 0.00243 nm

\[ \Delta \lambda = \frac{h}{m_e c} (1 - \cos \phi) \]

Electron Mass = \( 9.11 \times 10^{-31} \text{Kg} \)

Electron Charge = \( 1.602 \times 10^{-19} \text{C} \)

Atomic Mass Unit \( u = 1.6606 \times 10^{-27} \text{Kg} \) or 931.5 MeV/c^2

Proton Mass = \( 1.67 \times 10^{-27} \text{Kg} \) or 1.0073 u

deBroglie Wavelength \( \lambda = h/\rho \)

Electron Rest Energy = 0.511 MeV/c^2

Proton Rest Energy = 938 MeV/c^2

\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \]

\[ p = \frac{m}{\sqrt{1 - v^2/c^2}} \]

\[ E = \frac{m c^2}{\sqrt{1 - v^2/c^2}} = K - m c^2 \]

\[ E^2 = p^2 c^2 + m^2 c^4 \]

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Pl. write your answers neatly in your blue book, make sure your secret code number is written on all pages in indelible ink. Indicate the quiz # and date at the top of the page, before writing the solutions.

Pl. consult the proctor if you have difficulty understanding the question.
Problem 1 (10 points): Extreme Pool 1

A photon of energy 1.50 MeV has a head-on collision with a free electron at rest, and is scattered backwards through an angle $\phi = 180$ degrees. (a) Which way does the electron fly off with respect to the direction of incident photon? Draw a picture of this process (b) What is the wavelength of scattered photon? (c) What is the kinetic energy KE of the scattered electron? All physics here is relativistic.

Problem 2 (10 points): Particle Accelerators And The Act of Observation

To "observe" small objects, one measures the diffraction of particles (as in an electron microscope) whose de Broglie wavelength is approximately equal to the object's size. Find the kinetic energy (in eV) required for electrons to resolve (a) a large organic molecule of size $10^{-4}$m (b) atomic features of size $10^{-10}$m (c) a nucleus of size $10^{-14}$m. Repeat the calculation for a proton instead of an electron. (d) Which particle, originally at rest, requires a larger accelerating voltage?

Hint: Should you be using relativistic or non-relativistic formulae?
φ = 180°

$E_{ph} = 1.50 \text{ MeV}$

\[ \lambda_i = \frac{hc}{E} = \frac{4.136 \times 10^{-15} \text{ eV s}}{1.5 \times 10^6 \text{ eV}} = 8.27 \times 10^{-13} \text{ m} \]

\[ \Delta \lambda = \lambda_i (1 - \cos \phi) = (0.00243 \times 10^{-9} \text{ m})(1 - \cos 180°) \]
\[ = 4.88 \times 10^{-12} \text{ m} \]

\[ \lambda_f = \lambda_i + \Delta \lambda = 5.69 \times 10^{-12} \text{ m} \]

\[ \Delta E = h \nu_i - h \nu_f = hc \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_f} \right) \]
\[ = (4.136 \times 10^{-15} \text{ eV s})(3 \times 10^4 \text{ m/s}) \left( \frac{1}{8.27 \times 10^{-13} \text{ m}} - \frac{1}{5.69 \times 10^{-12} \text{ m}} \right) \]

\[ = 1.28 \text{ MeV} \]
\[ p = \frac{\lambda}{d} \quad \text{and} \quad d \Rightarrow p = \frac{n}{d} \]

(a) \[ d = 10^{-9} \text{ m} \]

\[ p = \frac{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}{10^{-9} \text{ m}} = \text{ mV} \]

\[ \text{electron} \quad m = 0.511 \text{ MeV} / c^2 \Rightarrow v \approx 7.3 \times 10^5 \text{ m/s} \]

\[ \frac{v}{c} \approx 0.02 < 10\% \]

\[ \text{non-relativistic OK} \]

\[ KE = \frac{p^2}{2m} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s} \times 10^9 \text{ m}^2)}{2 \times 661000 \text{ eV}} \]

\[ = 1.51 \text{ eV} \]

(b) \[ d = 10^{-10} \text{ m} \]

\[ p = \frac{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}{10^{-10} \text{ m}} = \text{ mV} \]

\[ \Rightarrow v \approx 7.3 \times 10^6 \text{ m/s} \]

\[ \frac{v}{c} \approx 0.02 < 10\% \]

\[ \text{non-relativistic OK} \]

\[ KE = \frac{p^2}{2m} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s}/\text{m})^2 c^2}{2 \times 571000 \text{ eV}} \]

\[ = 15.1 \text{ eV} \]

(c) \[ d = 10^{-14} \text{ m} \]

\[ p = \frac{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}}{10^{-14} \text{ m}} = \text{ mV} \]

\[ \Rightarrow \text{for electron} \quad v \approx 7.3 \times 10^{10} \text{ m/s} \]

\[ \text{impossible!} \]

\[ \text{must use relativistic} \]
\[ KE = E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2 = 124 \text{ MeV} \]

for proton \( m = 938 \text{ MeV}/c^2 \)

\[ P = \frac{4.136 \times 10^{-15} \text{ eV} \cdot s}{10^{-14} \text{ m}} = mv \implies v = 4.0 \times 10^7 \text{ m/s} \]

\( \implies \frac{v}{c} = 0.13 > 10\% \)

use relativistic

\[ KE = \sqrt{p^2c^2 + m^2c^4} - mc^2 = 8.17 \text{ MeV} \]

\[ KE = eV_0 \]

\[ V_0 = \frac{KE}{e} \implies \text{electron requires larger accelerating voltage} \]