3.10  

a) Let \( x \) be thickness of 1 card.

Then \( S^2 x = 0.59 \pm 0.005 \text{ in} \)

\[
\frac{1}{S^2} \Rightarrow x = 0.0113 \pm 0.0001 \text{ in}
\]

b) Let \( N \) be the minimum number of cards needed.

If error in 1 card's thickness = \( 2 \times 10^{-5} \text{ in} \)

error for \( N \) cards = \( N \times 2 \times 10^{-5} \geq 0.005 \text{ in} \)

\[
\Rightarrow N \geq 250 \quad \text{or 5 packs.}
\]

3.22  

\( I = 2.1 \pm 0.2 \text{ A} \), \( V = 1.02 \pm 0.01 \text{ V} \)

\( P = IV \); \( R = V/I \)

In both cases, fractional error

\[
\delta P \over P = \delta R \over R = \sqrt{(\delta I \over I)^2 + (\delta V \over V)^2} = 0.0137
\]

\[
\Rightarrow P = 2.14 \pm 0.03 \text{ W} \); \( R = 0.486 \pm 0.007 \Omega \)
\[ q = xy + \frac{x^2}{y} \quad ; \quad x = 6 \pm 0.1 \quad y = 3.0 \pm 0.1 \]

Cannot separate the variables \( x \) and \( y \) so must use

\[ (\delta q)^2 = \left( \frac{\delta q}{\delta x} \right)^2 + \left( \frac{\delta q}{\delta y} \right)^2 \]

\[ \frac{\delta q}{\delta x} = y + 2x = 7 \]

\[ \frac{\delta q}{\delta y} = x - \frac{x^2}{y^2} = 2 \]

Note: \( \frac{\delta q}{\delta x} \) is larger than \( \frac{\delta q}{\delta y} \), which tells us that the formula is more sensitive to changes in \( x \) than in \( y \).

Also note: \( \frac{\delta q}{\delta y} = x - \frac{x^2}{y^2} \), which means that an error in \( y \)'s contribution to term (1) is partially cancelled by the contribution to term (2).

Substituting: \( q = 6.3 + \frac{3.6}{3} = 30 \)

\[ (\delta q)^2 = (7.0.1)^2 + (2.0.1)^2 \]

\[ \Rightarrow \delta q = 0.7 \]

So \( q = 30 \pm 0.7 \)
4.16  We have \( \bar{g} = \frac{\Sigma g_i}{5} = 9.7 \text{m/s}^2 \)

Estimated standard deviation \( \sigma_g = \sqrt{\frac{\Sigma (g_i - \bar{g})^2}{N-1}} = 0.158 \text{m/s}^2 \)

\( \Rightarrow \) Standard error of mean \( \sigma_{\bar{g}} = \frac{\sigma_g}{\sqrt{N}} = 0.07 \)

So best estimate of \( g \) is \( \bar{g} = 9.7 \pm 0.07 \text{m/s}^2 \)

Compare with accepted \( g = 9.81 \text{m/s}^2 \)

\[ t = \frac{|g - \bar{g}|}{\sigma_{\bar{g}}} = \frac{|9.8 - 9.7|}{0.07} = 1.4 \text{ standard deviations} \]

So the result is consistent with the accepted value.

From Appendix A, Prob \( (|t| \leq 1.4) = 83.85\% \)

so we can accept this result at the 96.15\% confidence level.
4.23 \[ e = K \eta^{3/2} \]

So we need to find the small change \( \delta e \) due to a small systematic error \( \delta \eta \)

\[ \ln e = \ln K + \frac{3}{2} \ln \eta \]

Differentiate \[ \Rightarrow \frac{\delta e}{e} = \frac{3}{2} \frac{\delta \eta}{\eta} \]

So if \( \frac{\delta \eta}{\eta} = 0.4\% = 0.004 \), \[ \frac{\delta e}{e} = \frac{3}{2} \times 0.004 \]

\[ \Rightarrow \frac{\delta e}{e} = 0.6 \% \text{ fractional error.} \]

4.25 \[ u = f \lambda \text{ with } \lambda = 11.2 \pm 0.5 \text{cm, } f = 3000 \pm 30 \text{Hz (1\%)} \]

\[ \Rightarrow u = f \lambda = 3000 \times 0.112 \text{m} = 336 \text{ m/s} \]

Resulting uncertainty given by:

\[ (\frac{\delta u}{u})^2 = (\frac{\delta f}{f})^2 + (\frac{\delta \lambda}{\lambda})^2 \]

a) With \( \frac{\delta f}{f} = 0.1 \), \( \frac{\delta \lambda}{\lambda} = \frac{0.5}{11.2} = 0.0446 \Rightarrow \frac{\delta u}{u} = 0.046 \),

\[ \Rightarrow u = 336 \pm 15.4 \text{ m/s} \]

b) Now with \( \frac{\delta f}{f} = 0.03 \text{ (3\%)} \), \( \frac{\delta \lambda}{\lambda} = \frac{0.03}{11.2} = 0.009 \)

\[ \Rightarrow \frac{\delta u}{u} = 0.0313, \text{ dominated by error in } f. \]

\[ \Rightarrow u = 336 \pm (336 \times 0.0313) \text{ m/s} \]
\[ = 336 \pm 10.5 \text{ m/s} \]