Calculate the possible values of the $z$ component of angular momentum for an electron in a d subshell.

Sweet. So far I like this homework assignment.

All you need to know for this is spectroscopic notation, where:

- $s \rightarrow l = 0$  "sharp"  What those letters
- $p \rightarrow l = 1$  "principal"  If you were ever curious!
- $d \rightarrow l = 2$  "diffuse"
- $f \rightarrow l = 3$  "fuzzy"
- $g \rightarrow l = 4$  Go alphabetically from here
- $h \rightarrow l = 5$
  
  etc.

We are talking about a d subshell, which means $l = 2$. For a given $l$ value, $m_l$ can take on values

$$m_l = -l , -l+1 , \ldots , 0 , \ldots , l-1 , l$$

So for us,

$$m_l = -2 , -1 , 0 , 1 , 2$$

And the possible $L_z$ values are

$$L_z = m_l \times \hbar = \{ 2\hbar , \hbar , 0 , -\hbar , -2\hbar \}$$
#19. A hydrogen atom is in the 6g state.
   a) What is the principal quantum number?

   In this wacky spectroscopic notation, you write a state like
   \[ nl \]
   So 6g corresponds to
   \[ n = 6 \]
   \[ l = g \Rightarrow l = 4 \]

   \( n \) is the principal quantum number, so \[ n = 6 \]

   b) What is the energy of the atom?

   Using the Schrödinger equation, you get the same result for the hydrogen atom's energy as Bohr did, namely
   \[ E_n = \frac{-13.6\, \text{eV}}{n^2} \]

   So
   \[ E_6 = 0.378\, \text{eV} \]

   c) What are the values for the orbital quantum number (that would be \( l \)) and the magnitude of the electron's orbital angular momentum?
Good, these problems are great! For a given \( n \), \( \ell \) can take on the values

\[ \ell = 0, \ldots, n-1 \]

Since \( n = 6 \),

\[ \ell = 0, 1, 2, 3, 4, 5 \]

But they tell us we're in a \textit{bg} state, and the \( g \) means \[ \ell = 4 \]

and the magnitude of the electron's angular momentum is

\[ |\vec{L}| = \sqrt{\ell (\ell + 1)} \hbar = \sqrt{20} \hbar \]

d.) What are the possible values for the magnetic quantum number \( m_\ell \)?

For each value, find the corresponding \( z \) component of angular momentum and the angle that the angular momentum vector makes with the \( z \) axis.

Okey dokey. For a given \( \ell \),

\[ m_\ell = -\ell, -\ell+1, \ldots, 0, \ldots, \ell-1, \ell \]

So since \( \ell = 4 \),

\[ m_\ell = -4, -3, -2, -1, 0, 1, 2, 3, 4 \]
To find all this angular momentum garbage, remember that the picture we're talking about is

So we want to find the magnitude of $\vec{L}_z$ and $\theta$. We found in part a) that

$$|\vec{L}| = \sqrt{20} \, \hbar$$

and using trig, for a given $L_z$,

$$\theta = \arccos \left( \frac{L_z}{|\vec{L}|} \right)$$

Our $L_z$s are gonna be given by

$$L_z = m_y \, \hbar$$

So the possible values are

$$L_z = \pm 4 \, \hbar \quad \rightarrow \quad \theta = 0.464 \, \text{rad}$$

$$\pm 3 \, \hbar \quad \rightarrow \quad \theta = 0.835 \, \text{rad}$$

$$\pm 2 \, \hbar \quad \rightarrow \quad \theta = 1.11 \, \text{rad}$$

$$\pm \hbar \quad \rightarrow \quad \theta = 1.34 \, \text{rad}$$

$$0 \quad \rightarrow \quad \theta = \pm \frac{\pi}{2} \, \text{rad}$$

(- means measured from $z$ axis, ok?)
#20. Prove that the $n$th energy level of an atom has degeneracy equal to $n^2$.

Remember that degeneracy means that different states have the same energy. For a hydrogen-like atom the energy is

$$E_n = (-13.6 \text{ eV}) \frac{n^2}{\hbar^2}$$

So the only quantum number the energy depends on is $n$. But for every $n$, there's a buttload of other states, corresponding to different $\ell$ and $m_\ell$ values. And all of these will be degenerate. So our job is to count how many possible $\ell$ and $m_\ell$ values there are for a given $n$.

We know that for a given $n$,

$$\ell = 0, \ldots, n-1$$

So for the $n$th level, there are $n$ possible $\ell$ values. But for each $\ell$, we have more states given by all the different $m_\ell$s.

$$m_\ell = -\ell, \ldots, 0, \ldots, \ell$$

Or $2\ell+1$ possible $m_\ell$ values for a given $\ell$. So for $\ell = 0$,

$$m_\ell = 0$$

So there's only one state.

For $\ell = 1$

$$m_\ell = -1, 0, 1$$

So there's three states.
Similarly, for \( l=2 \), there will be 5 states, and so on. So for a given \( n \), we need to find all the \( l \)'s, see how many states each \( l \) gives us, and add them all up to find the total number of states:

\[
\text{total \# of states} = \sum_{l=0}^{n-1} (2l+1)
\]

So somehow we need to evaluate this sum for general \( n \). Now watch how clever your TA is.

We can write this sum out like

\[
\text{tot.\# of states} = 1 + 3 + 5 + 7 + \ldots + 2(n-2)+1 + 2(n-1)+1
\]

right? This is

\[
\text{tot.\# of states} = 1 + 3 + 5 + 7 + \ldots + 2n-3 + 2n-1
\]

Now watch this! It doesn't matter what order you add them in, so write the sum backwards

\[
\text{tot.\# of states} = 2n-1 + 2n-3 + \ldots + 7 + 5 + 3 + 1
\]

Now add the sum written forwards!

\[
\text{total \# of states} = 2n-1 + 2n-3 + \ldots + 7 + 5 + 3 + 1
\]
\[
+ \text{total \# of states} = 1 + 3 + \ldots + 2n-2 + 2n-5 + 2n-3 + 2n-1
\]
\[
2(\text{total \# of states}) = 2n + 2n + \ldots + 2n + 2n + 2n + 2n
\]

There's \( n \) of those terms, so

\[
2(\text{total \# of states}) = (2n)n = 2n^2
\]
So

\[
\text{total \# of states} = n^2 = \text{degeneracy!}
\]

Slick, no? (Actually, I think this should be \(2n^2\) due to spin degeneracy)

Actually, I ain't all that clever. I stole that trick from Gauss.

The story behind it is pretty cool. When Gauss was a little kid in grade school (about 8 or 9 years old or something), one day his class was being bad, so the teacher decided to punish them. He figured a good punishment would be to have them add all the integers from 1 to 100, but he figured it would take all day for them to do it and it would be painful and all that. But about 5 seconds after he told them to do it, Gauss yelled out "SOSO!" He came up with that trick when he was 9 years old, for God's sake! Supa-genius.

#22) Suppose that a hydrogen atom is in the 2s state. Taking \(r = a_0\), calculate values for

a. \(\psi_{2s}(a_0)\)

For the hydrogen atom, the states are in the form

\[
\psi_{nl}(a_0) = R_{n \ell}(r) \ Y_{\ell}^{m_{\ell}}(\theta, \phi)
\]

where the \(R_{n \ell}(r)\) are Laguerre polynomials and the \(Y_{\ell}^{m_{\ell}}(\theta, \phi)\) are the spherical harmonics,
For a given n, l, and m, you just go look them up.

They tell us our state is 2s

\[ \Rightarrow n = 2 \]
\[ l = 0 \Rightarrow m_l = 0 \]

So we know all our quantum numbers. We need to look up

\[ R_{2s}(r) = \left( \frac{1}{2a_o} \right)^{3/2} (2 - \frac{r}{a_o}) e^{-r/2a_o} \quad \text{(Table 7.4)} \]

\[ Y_0^0(\theta, \phi) = \frac{1}{\sqrt{4\pi}} \quad \text{(Table 7.3)} \]

So our state is

\[ \psi_{2s}(a_o) = R_{2s}(a_o) Y_0^0(\theta, \phi) \]

\[ = \frac{1}{\sqrt{4\pi}} \left( \frac{1}{2a_o} \right)^{3/2} e^{-r/2a_o} \]

\[ \psi_{2s}(a_o) = 9.88 \times 10^{-14} \text{ m}^{-3/2} \]

b.) \[ |\psi_{2s}(a_o)|^2 \]

Duh...

\[ |\psi_{2s}(a_o)|^2 = 9.75 \times 10^{-29} \text{ m}^2 \]

c.) \[ P_{2s}(a_o) \]

A little tougher...

\[ P_{2s}(a_o) = \int_0^{2\pi} \int_0^\pi |\psi_{2s}(a_o)|^2 a_o^2 \sin \theta \, d\theta \, d\phi \]
#22. Continued

\[ P_{2s}(r_0) = 4\pi \frac{1}{2} \left( a_0 \right)^2 a_0^2 \]

\[ = 3.48 \times 10^{10} \text{ m}^{-1} \]

#23. The radial part of the wavefunction for the hydrogen atom in the 2p state is given by

\[ R_{2p}(r) = A r e^{-r/2a_0} \]

where \( A \) is a constant and \( a_0 \) is the Bohr radius. Using this expression, calculate the average value of \( r \) for an electron in this state.

We need to calculate \( \langle r \rangle \) for this thing. That'll be

\[ \langle r \rangle = \int_0^{\infty} \int_0^{\pi} \int_0^{2\pi} \psi^*(r,\theta,\phi) \psi(r,\theta,\phi) r^2 \sin \theta \, dr \, d\theta \, d\phi \]

\[ = \int_0^{\infty} dr \, r^3 |R_{2p}(r)|^2 \int_0^{\pi} \sin \theta \, d\theta \int_0^{2\pi} d\phi \int_0^{\infty} \Phi(r) \, dr \]

The integrals of \( \theta \) and \( \phi \) will both be equal to one since \( \Phi(\theta) \) and \( \Phi(\phi) \) are individually normalized. So we're left with

\[ \langle r \rangle = \int_0^{\infty} r^3 |R_{2p}(r)|^2 \, dr \]

\[ = A^2 \int_0^{\infty} r^5 e^{-r/2a_0} \, dr \]

You can go look this mother up, or if you like pain, you can integrate by parts 5 times! Not for me!
#23) continued

Look it up and get

\[
\langle r \rangle = A^2 a_o^6 5! = 5a_o = 2.695 \text{ Å}
\]

#25) Calculate the average potential and kinetic energies for the electron in the ground state of hydrogen.

The potential energy is

\[
U(r) = -\frac{ke^2}{r}
\]

So the average is (again, we only gotta worry about the radial integral)

\[
\langle U(r) \rangle = \int_0^{\infty} \psi^*_r \psi \left( -\frac{ke^2}{r} \right) r^2 dr
\]

\[
= -\frac{4ke^2}{a_o^3} \int_0^{\infty} e^{-2r/a_o} dr
\]

You can integrate by parts to get

\[
\langle U(r) \rangle = -\frac{ke^2}{a_o} = -2.192 \text{ eV}
\]

To get \( \langle K \rangle \),

\[
\langle K \rangle + \langle U(r) \rangle = \langle E \rangle = -13.6 \text{ eV}
\]

\( \therefore \langle K \rangle = 13.6 \text{ eV} \)
Compute the probability that a 2s electron of hydrogen will be found inside the Bohr radius for this state, \( a_0 \). Compare this with the probability of finding a 2p electron in the same region.

We need to look up \( \psi_{2s}(r, \theta, \phi) \)

\[
\psi_{2s}(r, \theta, \phi) = \left( \frac{1}{2a_0} \right)^{3/2} \left( 2 - \frac{r}{a_0} \right) e^{-r/2a_0} \gamma_0(\theta, \phi)
\]

Again the \( \theta \) and \( \phi \) integrals will give 1, so

\[
P_{2s}(r) = \int_0^{a_0} |R_{2s}(r)|^2 r^2 dr
\]

\[
= \left( \frac{1}{2a_0} \right)^3 \int_0^{a_0} \left( 2 - \frac{r}{a_0} \right)^2 e^{-r/a_0} r^2 dr
\]

\[
= \frac{1}{8a_0} \int_0^{a_0} (4 - 4 \frac{r}{a_0} + \frac{r^2}{a_0^2}) \frac{r^2}{a_0^2} e^{-r/a_0} dr
\]

Let \( u = \frac{r}{a_0} \quad \text{when} \quad r = 0, \ u = 0 \)

\( du = \frac{dr}{a_0} \quad \text{when} \quad r = a_0, \ u = 1 \)

\[
= \frac{1}{8} \int_0^1 (4 u^2 - 4 u^3 + u^4) e^{-u} du
\]

Good God! Need to integrate each of these terms by parts repeatedly to get

\[
= \frac{1}{8} \left[ (4 u^3 - 4 u^2 - u^4) + (12 u^2 - 4 u^3 - 8 u) + (24 u - 12 u^2 - 8) 
\right.
\]

\[
+ \left. (24 - 24 u) - 24 \right] e^{-u} \bigg|_0^1
\]
#27) continued

Putting in limits

\[ P_{2s}(r) = 0.176 \]

Do the same thing again for \( P_{2p}(r) \)

\[ P_{2p}(r) = \left( \frac{1}{2a_0} \right)^{3/2} \frac{r}{13a_0} e^{-r/12a_0} \]

and you'll wind up with

\[ P_{2p}(r) = 0.371 \]

#30) Calculate the uncertainty product \( \Delta r \Delta p \) for the 1s electron of a hydrogen-like atom with atomic number \( Z \).

To find uncertainties, remember

\[ \Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} \]

\[ \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \]

\( \Delta p \) is easiest. \( \langle p \rangle = 0 \) due to symmetry, and we can find \( \langle p^2 \rangle \) using

\[ \langle 1s \rangle = \frac{\langle p^2 \rangle}{2m} = |\langle E \rangle| = \frac{Z^2 \hbar^2}{2m a_0^2} \]

\[ \therefore \langle p^2 \rangle = \left( \frac{Z \hbar}{a_0} \right)^2 \Rightarrow \Delta p = \frac{Z \hbar}{a_0} \]
Now comes the painful part. We have to find \( \langle r \rangle \) and \( \langle r^2 \rangle \). Poop.

\[
\langle r \rangle = \int_0^\infty \rho_n^*(r) r \rho_n(r) r \, dr
\]

\[
= \left( \frac{2}{a_0} \right)^3 4 \int_0^{\infty} r e^{-2r/a_0} \, dr
\]

Integrate by parts, of course.

\[
= \frac{3a_0}{2a_0}
\]

\[
\langle r^2 \rangle = \left( \frac{2}{a_0} \right)^3 4 \int_0^{\infty} r^4 e^{-2r/a_0} \, dr
\]

Again, integrate by parts (4 times!)

\[
= \frac{3a_0^2}{2a_0^2}
\]

So

\[
\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} = \sqrt{\frac{3a_0^2}{2a_0^2} - \frac{9a_0^2}{4a_0^2}}
\]

\[
= \sqrt{\frac{3a_0^2}{4a_0^2}} = \sqrt{\frac{3}{4}} \frac{a_0}{a_0} = \frac{\sqrt{3}}{2}
\]

\[
\Delta r \Delta \rho = \sqrt{\frac{3}{4}} \frac{a_0}{a_0} \left( \frac{2\hbar}{a_0} \right) = \sqrt{\frac{3}{4}} \frac{\hbar}{k}
\]
Chapter 8

1. In the technique known as electron spin resonance (ESR), a sample containing unpaired electrons is placed in a magnetic field. Consider the simplest situation, that in which there is only one electron and therefore only two possible energy states, corresponding to \( m_s = \pm \frac{1}{2} \). In ESR, the electron's spin magnetic moment is "flipped" from a lower energy state to a higher energy state by the absorption of a photon. (The lower energy state corresponds to the case in which the magnetic moment \( \vec{\mu}_s \) is aligned with the magnetic field, and the higher energy state corresponds to the case when \( \vec{\mu}_s \) is aligned against the field.) What is the photon frequency required to excite an ESR transition in a magnetic field of 0.35 T?

We need

\[
\vec{\mu}_s = \frac{g_\text{\textsubscript{\mu}}}{2m_e} \vec{S} \quad \text{(eq. 8.9)}
\]

Now let's say our magnetic field is pointing in the +z direction, so that

\[
\vec{B} = B\hat{z}
\]

The energy of the electron in the field is given by equation 8.10:

\[
U = -\vec{\mu} \cdot \vec{B}
\]
#1.) continued

Since $\hat{B}$ is only in the $z$ direction,

$$U = \pm \frac{1}{2} m_e B = \pm \frac{g \gamma S_z B}{2 m_e} \quad (+ \text{ for anti-aligned, } - \text{ for aligned})$$

So the aligned state has energy

$$U_\parallel = - \frac{g \gamma S_z B}{2 m_e} = - \frac{g}{2 m_e} \left( \frac{1}{2} \hbar \right) B$$

$$= - \frac{g \frac{\hbar}{4 m_e}}{4 m_e} B$$

And the anti-aligned state has energy

$$U_\perp = g \frac{\hbar}{4 m_e} B$$

The difference in energy of the two states is

$$\Delta U = U_\perp - U_\parallel = \frac{g \frac{\hbar}{2 m_e}}{2 m_e} B$$

So to excite the electron from aligned to anti-aligned, we have to hit it with a photon of that energy

$$\Rightarrow E_\gamma = \Delta U = \hbar f$$

$$\therefore \quad f = \frac{\Delta U}{\hbar} = \frac{g \frac{\hbar}{2 m_e}}{2 m_e} B = \frac{g}{4 \hbar m_e} B$$

$$= 9.79 \times 10^9 \text{ Hz}$$

for $e^{-}, g = 2$
#5) The force on a magnetic moment \( \vec{\mu} \) in a nonuniform field \( B_z \) is given by

\[
F_z = \mu_0 \frac{dB_z}{dt}
\]

If a beam of silver atoms travels a horizontal distance of 1 m through such a field and each atom has a speed of 100 m/s, how strong must the field gradient \( dB_z/dt \) be in order to deflect the beam 1 mm?

This is just a 2A problem. The atom experiences a force

\[
F_z = \mu_0 \frac{dB_z}{dt} = ma
\]

\[
\Rightarrow a = \frac{\mu_0}{m} \frac{dB_z}{dt}
\]

\[
\therefore v_z = \frac{\mu_0}{m} \frac{dB_z}{dt} + c
\]

assuming \( v_{z0} = 0 \),

\[
v_z = \frac{\mu_0}{m} \frac{dB_z}{dt}
\]

\[
\therefore z = \frac{\mu_0}{2m} \frac{dB_z}{dt} t^2 + c
\]

let \( z_0 = 0 \).
\[ z = \frac{\mu_e}{2m} \frac{dB_z}{dt} t^2 \]

Now, the atoms are travelling horizontally 1 m at a speed \( v_x = 100 \text{ m/s} \)

\[ v_x \frac{d}{dt} t \Rightarrow t = \frac{d}{v_x} \]

\[ \therefore z = \frac{\mu_e}{2m} \frac{dB_z}{dt} \left( \frac{d}{v_x} \right)^2 \]

which we want to be 1 mm. So

\[ \frac{dB_z}{dt} = \frac{2m \varepsilon}{\mu_e} \left( \frac{v_x}{d} \right)^2 \]

where \( z = 1 \times 10^{-3} \text{ m} \)

\[ d = 1 \text{ m} \]

\[ v_x = 100 \text{ m/s} \]

All we need now is \( \mu_e \), which is due to an unpaired electron's spin

\[ \therefore \mu_e = \frac{9.4}{2m_e} \]

\[ S_\varphi = \frac{e \hbar}{2m_e} \]

So

\[ \frac{dB_z}{dt} = 0.387 \text{ T/m} \]
#7.) When the idea of electron spin was introduced, the electron was thought to be a tiny charged sphere. Find the equatorial speed under the assumption that the electron is a uniform sphere of radius $3 \times 10^{-6}$ nm, and compare with c.

The magnitude of the spin angular momentum is

$$|\vec{S}| = \sqrt{S(S+1)} \hbar$$

$$\sqrt{\frac{1}{2}(\frac{1}{2}+1)} \hbar = \frac{\sqrt{3}}{2} \hbar$$

which, since it's angular momentum, also has to be

$$|\vec{S}| = I\omega$$

where $I$ is the moment of inertia and $\omega$ is the angular velocity. For a uniform sphere,

$$I = \frac{2}{5} MR^2$$

$$|\vec{S}| = \frac{2MR^2\omega}{5}$$

$$\omega = \frac{5|\vec{S}|}{2MR^2} = \frac{V_{eq}}{R}$$

$$V_{eq} = \frac{5|\vec{S}|}{2MR} = \frac{5\sqrt{3} \hbar}{4 MR} = 83.5 \times 10^9 \text{ m/s}$$

$$V_{eq} = 2.78 \times c$$

**Fat chance!**
Starting with the expression $\mathbf{J} = \mathbf{L} + \mathbf{S}$ for the total angular momentum of an electron, derive an expression for the scalar product $\mathbf{L} \cdot \mathbf{S}$ in terms of the quantum numbers $j, l, \text{and } s$.

**Easy:**

$\mathbf{J} = \mathbf{L} + \mathbf{S}$

$\implies |\mathbf{J}|^2 = \mathbf{J} \cdot \mathbf{J} = (\mathbf{L} + \mathbf{S}) \cdot (\mathbf{L} + \mathbf{S})$

$= |\mathbf{L}|^2 + |\mathbf{S}|^2 + 2 \mathbf{L} \cdot \mathbf{S}$

$2 \mathbf{L} \cdot \mathbf{S} = |\mathbf{J}|^2 - |\mathbf{L}|^2 - |\mathbf{S}|^2$

$\mathbf{L} \cdot \mathbf{S} = \frac{1}{2} (|\mathbf{J}|^2 - |\mathbf{L}|^2 - |\mathbf{S}|^2)$

$\therefore \mathbf{L} \cdot \mathbf{S} = \frac{1}{2} (j(j+1) \hbar^2 - l(l+1) \hbar^2 - s(s+1) \hbar^2)$

b) Using $\mathbf{L} \cdot \mathbf{S} = |\mathbf{L}||\mathbf{S}| \cos \theta$, where $\theta$ is the angle between $\mathbf{L}$ and $\mathbf{S}$, find the angle between $\mathbf{L}$ and $\mathbf{S}$ for the following states:

1) $P_{1/2}$

Stupid spectroscopic notation. This is in the form $l_j$.

So $l = 1$, $j = \frac{1}{2}$. Since

$\mathbf{J} = \mathbf{L} + \mathbf{S}$

$\implies s = \frac{1}{2}$

$|\mathbf{L}|^2 = l(l+1) \hbar^2 = 2 \hbar^2$

$|\mathbf{S}|^2 = s(s+1) \hbar^2 = \frac{3}{4} \hbar^2$

$|\mathbf{J}|^2 = j(j+1) \hbar^2 = \frac{3}{4} \hbar^2$.
\[ \vec{\mathbf{S}} = \frac{1}{2} \left( j(j+1) \hbar^2 - \ell(\ell+1) \hbar^2 - s(s+1) \hbar^2 \right) \]

\[ \therefore \theta = \arccos \left[ \frac{j(j+1) \hbar^2 - \ell(\ell+1) \hbar^2 - s(s+1) \hbar^2}{2\sqrt{j(j+1) \hbar^2 \sqrt{s(s+1) \hbar^2}}} \right] \]

\[ \theta = 144.7^\circ \]

\[ P_{3/2} \implies \ell = 1, \quad j = \frac{3}{2}, \quad s = \frac{1}{2} \]

\[ \therefore \theta = 65.9^\circ \]

2) \[ H_{9/2} \implies \ell = 5, \quad j = \frac{9}{2}, \quad s = \frac{1}{2} \]

\[ \therefore \theta = 129.2^\circ \]

\[ H_{11/2} \implies \ell = 5, \quad j = \frac{11}{2}, \quad s = \frac{1}{2} \]

\[ \theta = 58.2^\circ \]
#15) Spin-Orbit Energy in an Atom - Estimate the magnitude of the spin-orbit energy for an atomic electron in the hydrogen 2p state.

(Hint: from the vantage point of the electron, the nucleus appears to circle it in an orbit with radius equal to the Bohr radius for this state. Treat the orbiting nucleus as a current in a circular wire loop and use the result from classical electromagnetism.

\[ B = \frac{2 \kappa \mu}{r^3} \]

for the B field at the center of the loop with radius r and magnetic moment \( \mu \). Here \( \kappa = 10^{-9} \text{ N} \cdot \text{mA}^{-2} \).

We usually think of the electron as orbiting the nucleus:

![Diagram of electron orbiting nucleus]

But if you were sitting on the electron, it would look like the nucleus was orbiting you (just like it looks like the sun is orbiting the Earth from here, even though it's the other way around, in reverse):

![Diagram of nucleus orbiting electron]
This is a charge going in a circle, which is a current loop. They tell us the magnetic field generated by this is

\[ B = \frac{2k_m a}{r^3} \]

and

\[ \mu = \frac{e}{2m_e} L = \frac{eh}{2m_e} \sqrt{l(l+1)} \]

For a p state \( l=1 \), so

\[ \mu = \frac{\sqrt{2} e h}{2m_e} \]

The Bohr radius of a 2p orbit is \( 4a_0 \), so

\[ B = \frac{2k_m}{(4a_0)^3} \left( \frac{\sqrt{2} e h}{2m_e} \right) = 0.2960 \text{T} \]

The energy due to the spin interaction with this field is

\[ U = -\vec{\mu} \cdot \vec{B} = \pm \frac{e}{2m_e} \| \vec{B} \| \]

\[ U = \pm 1.59 \times 10^{-5} \text{eV} \]