#4.) Let frame $K$ be the frame in which $m_1 = 0.3$ kg, $u_{10} = 5$ m/s, $m_2 = 0.2$ kg, $u_{20} = 3$ m/s

Because momentum is conserved, we know

$$\vec{P}_{i0} + \vec{P}_{z0} = \vec{P}_{if} + \vec{P}_{zf}$$

$$\therefore m_1 u_{10} + m_2 u_{20} = m_1 u_{1f} + m_2 u_{2f}$$(1)

Let frame $K'$ be the frame with velocity $V = 2$ m/s relative to $K$

Because $u_{10}, u_{20} \ll c$, transform coordinates using Galilean transformation (eq. 1.1):

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$$\Rightarrow \quad x = x' + vt$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

$$\therefore \quad u_x = u_x' + v$$
$$u_y = u_y'$$
$$u_z = u_z'$$

Plug these transformations into equation (1):

$$m_1 (u_{10} + v) + m_2 (u_{20} + v) = m_1 (u_{1f} + v) + m_2 (u_{2f} + v)$$

$$(m_1 u_{10} + m_2 u_{20}) + (m_1 + m_2)v = (m_1 u_{1f} + m_2 u_{2f}) + (m_1 + m_2)v$$

$$\therefore m_1 u_{10} + m_2 u_{20} = m_1 u_{1f} + m_2 u_{2f}$$

$$\Rightarrow \quad \vec{P}_{i0} + \vec{P}_{z0} = \vec{P}_{if} + \vec{P}_{zf}$$
\( \# 9 \) With what speed will a clock have to be moving in order to run at a rate that is one half the rate of a clock at rest?

Use Lorentz time transformation (eq. 1.28 or 1.30)

\[
t' = \gamma \left( t - \frac{vx}{c^2} \right)
\]

If we measure a length of time at with the clock at rest, we require

\[
\Delta t' = 2 \Delta t
\]

\[
t_f' - t_o' = \gamma \left( t_f - \frac{vx}{c^2} \right) - \gamma \left( t_o - \frac{vx}{c^2} \right)
\]

\[
\Delta t' = \gamma \Delta t
\]

\[
2 \Delta t = \gamma \Delta t
\]

\[
\Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
1 - \frac{v^2}{c^2} = \frac{1}{4}
\]

\[
\therefore \quad v = \sqrt{\frac{3}{4}} \text{ m/s} = 0.866 \text{ m/s}
\]

\( \# 9 \) A meter stick moving in a direction parallel to its length appears to be only 75 cm long to an observer. What is the speed of the meter stick relative to the observer?

At rest, the meter stick is observed to look like

\[
\Delta x = x_2 - x_1 = 1 \text{ m}
\]

When moving with relative velocity \( v \), it appears to look like

\[
\Delta x = x_2' - x_1' = 0.75 \text{ m} = 0.75 \Delta x'
\]
#9) continued

Use Lorentz transformation (eq. 1.25)

\[ x' = \gamma (x - vt) \]
\[ x'_2 - x'_1 = \gamma (x'_2 - vt) - \gamma (x'_1 - vt) \]

\[ \Delta x' = \gamma \Delta x \]

Lorentz contraction equation

(remember \( \Delta x' \) is proper length!)

\[ \Delta x' = \gamma (0.75) \Delta x' \]

\[ \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{0.75} \]

\[ 1 - \frac{v^2}{c^2} = (0.75)^2 \]

\[ \therefore v = \sqrt{1 - (0.75)^2} c = 0.661 c \]

#10.) A spacecraft moves at a speed of 0.900 c. If its length is \( L \) as measured
by an observer on the spacecraft, what is its length measured by a ground
observer.

Another Lorentz contraction problem: Use equation (3)

\[ \Delta x' = \gamma \Delta x \]
\[ \Delta x' = L \]

\[ \Rightarrow \Delta x = \frac{L}{\gamma} = L \sqrt{1 - \frac{v^2}{c^2}} \]

; \( v = 0.900 \) c

\[ \Delta x = 0.436 \ L \]
#13.) An astronaut at rest on Earth has a heartbeat rate of 70 beats/min. What will this rate be when the astronaut is traveling in a spaceship at 0.90c as measured a.) by an observer also in the ship?

The observer is at rest relative to the astronaut.

:: rate is still 70 beats/min

b.) by an observer at rest on the Earth?

if heart rate is 70 beats/min, interval between beats is

\[ \Delta t = \frac{1}{70} \text{min/beat} \]

Use time dilation equation (2):

\[ \Delta t' = \gamma \Delta t \]

\[ \Rightarrow \Delta t' = \Delta t \frac{\gamma}{\sqrt{1 - \frac{v^2}{c^2}}} = 0.0328 \text{min/beat} \]

:: rate is 30.5 beats/min

#15.) A rod of length \( L_0 \) moves with speed \( v \) along the horizontal direction. The rod makes an angle of \( \Theta_0 \) with respect to the \( x' \) axis.

a.) Show that the length of the rod as measured by a stationary observer is given by

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2 \Theta_0} \]

The length of the rod can be written as

\[ L_0 = L_0^x + L_0^y \]

\[ = L_0^2 \cos^2 \Theta_0 + L_0^2 \sin^2 \Theta_0 \]

If the rod moves with velocity \( v \) in \( x \) direction, \( L_0^x \) will Lorentz contract (use eq. (3)), \( L_0^y \) will remain unaffected.
Lorentz contraction

\[ \Delta x' = \gamma \Delta x \] (again, don't forget \( \Delta x' \) is proper length. \( \Delta x = L_0 x \))

\[ L_0 \cos \Theta_0 = \gamma \Delta x \]

\[ \Rightarrow \Delta x = \frac{L_0 \cos \Theta_0}{\gamma} = L_0 \cos \Theta_0 \sqrt{1 - \frac{v^2}{c^2}} \]

To observer at rest, the length of the rod will appear to be

\[ L^2 = \Delta x^2 + L_0^2 \]

\[ = L_0^2 \cos^2 \Theta_0 (1 - \frac{v^2}{c^2}) + L_0^2 \sin^2 \Theta \]

\[ = L_0^2 (\sin^2 \Theta_0 + \cos^2 \Theta_0) - L_0^2 \frac{v^2}{c^2} \cos^2 \Theta_0 \]

\[ = L_0^2 (1 - \frac{v^2}{c^2} \cos^2 \Theta_0) \] (recall \( \sin^2 \Theta + \cos^2 \Theta = 1 \))

\[ \therefore L = L_0 \sqrt{1 - \frac{v^2}{c^2} \cos^2 \Theta_0} \]

b) Show that the angle that the rod makes with the x axis is given by the expression

\[ \tan \Theta = \gamma \tan \Theta_0 \]

In the rod frame

\[ \tan \Theta_0 = \frac{L_0 y}{L_0 x} \]

To the stationary observer,

\[ \tan \Theta = \frac{L_0 y}{\Delta x} \]

From the last part,

\[ \Delta x = \frac{L_0 x}{\gamma} \]

\[ \therefore \tan \Theta = \frac{L_0 y}{L_0 x / \gamma} = \frac{L_0 y}{L_0 x} = \gamma \tan \Theta_0 \]

\[ \therefore \]
The classical Doppler shift for light: A light source recedes from an observer with speed \( v \) that is small compared with \( c \).

a) Show that in this case, Equation 1.13 reduces to

\[
\frac{\Delta f}{f} = -\frac{v}{c}
\]

Equation 1.13 is

\[
f_{\text{obs}} = \frac{\sqrt{1 + \frac{v}{c}}}{\sqrt{1 - \frac{v}{c}}} f_{\text{source}} \rightarrow \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}} f_{\text{source}} \quad \text{for receding source}
\]

\[
\Delta f = f_{\text{obs}} - f_{\text{source}} = \left[ \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}} - 1 \right] f_{\text{source}}
\]

\[
\Rightarrow \frac{\Delta f}{f} = -\left[ 1 - \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}} \right]
\]

\[
= \left[ 1 - \left( 1 - \frac{v}{c} \right)^{1/2} \left( 1 + \frac{v}{c} \right)^{-1/2} \right]
\]

Taylor expand the two radicals (recall \( f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n f(x)}{dx^n} |_{x=0} x^n \) )

\[
(1 - \frac{v}{c})^{1/2} = (1 - x)^{1/2} = 1 - \frac{x}{2} - \frac{x^2}{8} - \ldots \quad (x = \frac{v}{c})
\]

\[
(1 + \frac{v}{c})^{-1/2} = (1 + x)^{-1/2} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \ldots
\]

If \( v \ll c \), \( \frac{v}{c} \ll 1 \) \( \Rightarrow \) ignore terms of order \( (\frac{v}{c})^2 \) and higher

\[
\therefore (1 - x)^{1/2} \approx 1 - \frac{x}{2}
\]

\[
(1 + x)^{-1/2} \approx 1 - \frac{x}{2}
\]

\[
\Rightarrow (1-x)^{1/2}(1+x)^{-1/2} \approx (1 - \frac{x}{2})^2 = 1 - X + \left( \frac{x^2}{4} \right) = 0
\]

\[
\therefore \frac{\Delta f}{f} \approx -\left[ 1 - (1 - x) \right] = -X = -\frac{v}{c}
\]
(a) continued

b) Also show that in this case
\[ \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c} \]

\[ c = \lambda f \]
\[ \lambda = \frac{\xi}{f}, \quad f = \frac{\xi}{\lambda} \]
\[ \Rightarrow \Delta \lambda = - \frac{\xi}{f^2} \Delta f \]
\[ \frac{f}{c} \frac{\Delta \lambda}{\lambda} = - \frac{\Delta f}{f} \]
\[ \therefore \] \[ \frac{\Delta \lambda}{\lambda} \approx - \frac{\Delta f}{f} \frac{v}{c} \]

\[ \Delta \lambda \approx \frac{\xi}{f} \frac{v}{c} \]

\[ \Box \]

c) Spectroscopic measurements of an absorption line normally found at \( \lambda = 397 \text{ nm} \) reveal a red shift of 20 nm for light coming from a galaxy in Ursa Major. What is the recessional speed of this galaxy?

Assume \( v \ll c \). Then

\[ \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c} \]
\[ v = c \frac{\Delta \lambda}{\lambda}; \quad \Delta \lambda = 20 \text{ nm}, \quad \lambda = 397 \text{ nm} \]

\[ v = 1.5 \times 10^8 \text{ m/s} \]
#9. Doppler radar. An important practical application of the Doppler effect is the use of radar to determine the speed of a moving object. In this case, the Doppler shift of the electromagnetic radar signal reflected from the moving object is directly proportional to the radial speed of the moving object with respect to the radar transmitter. If a police radar transmitter radiates at 10.0 GHz, calculate the frequency shift observed by the police for a car traveling at

a.) 60.0 mi/h

Use the Doppler shift equation for approaching source (eq. 1.13)

Initially, consider the radar gun as source, the car as the observer. The car will receive the radar signal with frequency

$$ f_{\text{car}} = \sqrt{1 + \frac{v}{c}} f_{\text{rad}} \sqrt{1 - \frac{v}{c}} $$

The car will reflect the signals at this frequency. However, since the car is moving, the radar receiver will see them Doppler shifted again

$$ f'_{\text{rad}} = \sqrt{1 + \frac{v}{c}} f_{\text{car}} \sqrt{1 - \frac{v}{c}} $n$$

$$ = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} f_{\text{rad}} (1 + \frac{v}{c}) (1 - \frac{v}{c})^{-1} f_{\text{rad}} $$

Just like the last problem, v << c, so we can Taylor expand and simplify

$$ (1 - \frac{v}{c})^{-1} = (1-x)^{-1} = 1 + x + x^2 + ... \quad x = \frac{v}{c} $$

$$ \approx 1 + x $$

$$ \therefore f'_{\text{rad}} \approx (1 + x)^2 f_{\text{rad}} = (1 + 2x + x^2) f_{\text{rad}} $$

$$ \approx (1 + 2x) f_{\text{rad}} $$
\[ \Delta f = f_{\text{rad}} - f_{\text{rad}} = 2 \times f_{\text{rad}} \]

\[ \therefore \frac{\Delta f}{f_{\text{rad}}} = 2 \times \frac{2v}{c} \]

\[ \Delta f = \frac{2v}{c} f_{\text{rad}} \]  

(4)

Make sure your units jive!

\[ \Delta f = 1790 \text{ Hz} \]

b.) 70.0 mi/h

Use eq. (4) to get

\[ \Delta f_2 = 2090 \text{ Hz} \]

c.) The radar must be able to distinguish between the two cars, and therefore it must be sensitive enough to distinguish between the two frequency shifts \( \Delta f_1 + \Delta f_2 \)

\[ \Delta f = \Delta f_2 - \Delta f_1 = 300 \text{ Hz} \]

\[ \therefore \frac{\Delta f}{1 \times 10^{10}} = 3 \text{ parts in } 10^8 \]
#20. Two spaceships approach each other, each moving with the same speed as measured by an observer on the Earth. If their relative speed is \(0.70\ c\), what is the speed of each spaceship?

Use Lorentz transformation equation

for velocities (eq. 1.32)

\[ u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \]

In this case,

\[ u'_B = \frac{u_B - u_A}{1 - \frac{u_B u_A}{c^2}} = -\frac{2u_A}{1 + \frac{u_A^2}{c^2}} \]

\[ (1 + \frac{u_A^2}{c^2})u_B' = -2u_A \]

\[ \frac{u_B'}{c^2} u_A^2 + 2u_A + u_B' = 0 \]

Solve using quadratic equation

\[ u_A = -2 \pm \sqrt{4 - 4\left(\frac{u_B'}{c^2}\right)\left(\frac{u_B'}{c^2}\right)} \frac{c^2}{2\left(\frac{u_B'}{c^2}\right)} = \left[ -1 \pm \sqrt{1 - \frac{u_B^2}{c^2}} \right] \]

\[ u_A' = -0.41\ c \]

\[ \therefore u_B = 0.41\ c \]
#22.) An observer on Earth observes two spacecraft moving in the same direction toward the Earth. Spacecraft A appears to have a speed of 0.50 c, and spacecraft B appears to have a speed of 0.80 c. What is the speed of spacecraft A measured by an observer in spacecraft B?

In other words, they are asking for the relative speed. The picture looks like:

Use the velocity addition formula again (eq. 1.32)

\[
U_x' = \frac{U_x - V}{1 - \frac{U_x V}{c^2}}
\]

\[
U_A' = \frac{U_A - U_B}{1 - \frac{U_A U_B}{c^2}} = -0.5 \text{ c}
\]
An observer in frame $S$ sees lightning simultaneously strike two points 100 m apart. The first strike occurs at $x_1=y_1=z_1=t_1=0$ and the second at $x_2=100\text{ m}, y_2=z_2=t_2=0$.

(a) What are the coordinates of these two events in a frame $S'$ moving in the standard configuration (v along +x axis) at $0.70\ c$ relative to $S$?

Just need to Lorentz transform $S$ coordinates into $S'$ coordinates using eqs. 1.25-1.29:

\[
x' = y(x-vt)
\]
\[
y' = y
\]
\[
z' = z
\]
\[
t' = t - \frac{v x}{c^2}
\]
\[
y' = (1 - \frac{v^2}{c^2})^{-1/2} = 1.40
\]

\[
\therefore x_1' = 0 \quad x_2' = y(x_2-vt_2) = 140\text{ m}
\]
\[
y_1' = 0 \quad y_2' = 0
\]
\[
z_1' = 0 \quad z_2' = 0
\]
\[
t_1' = 0 \quad t_2' = t - \frac{v x_2}{c^2} = -0.33\text{ ms}
\]

(b) How far apart are the events in $S'$?

Easy.

\[
\Delta x' = x_2' - x_1' = 140\text{ m}
\]

(c) $t_2' \neq t_1'$, so the events are not simultaneous. The difference is

\[
\Delta t' = t_2' - t_1' = -0.33\text{ ms} \Rightarrow \text{Event 2 occurs first by 0.33 ms}
\]
As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by an Earth observer to have velocity \( u_y = 0.9c \) and B to have velocity \( u_x = 0.9c \), find the speed of ship A as measured by the pilot of B.

This is a little more complicated. The picture is:

We need to Lorentz transform both \( x \) and \( y \) components of ship A's velocity using eqs 1.32 and 1.33.

\[
\begin{align*}
\mathbf{u}' &= \mathbf{u} - \mathbf{v} \\
&= \frac{\mathbf{u} - \mathbf{v}}{1 - \frac{\mathbf{u} \cdot \mathbf{v}}{c^2}} \\
\Rightarrow \quad \mathbf{u}_x' &= \frac{u_{Ax} - u_{Bx}}{1 - \frac{u_{Ax}u_{Bx}}{c^2}} = -u_{Bx} = -0.90c \\
\mathbf{u}_y' &= \frac{u_{Ay} - u_{By}}{1 - \frac{u_{Ay}u_{By}}{c^2}} \\
&= \frac{u_{Ay} - u_{By}}{\gamma \left(1 - \frac{u_{Ay}u_{By}}{c^2}\right)} = -0.392c \\
\end{align*}
\]

The speed of A seen by B is

\[
V_A = \sqrt{u_{Ax}'^2 + u_{Ay}'^2} = 0.982c
\]
#26.) Calculate the momentum of a proton moving with a speed of

a.) 0.010c

Just use eq. 1.35

\[ p = \gamma m_0 = \frac{m_0 u}{\sqrt{1 - \frac{u^2}{c^2}}} \]

Plug and chug. You can find the proton mass on the inside front cover as

\[ m_p = 1.673 \times 10^{-27} \text{ kg} \]
\[ = 938.3 \text{ MeV/c}^2 \]

This gives

\[ p = 5.01 \times 10^{-21} \text{ kg m/s} \]

b.) 0.50c

\[ p = 2.89 \times 10^{-19} \text{ kg m/s} \]

c.) 0.90c

\[ p = 1.03 \times 10^{-18} \text{ kg m/s} \]

d.) You can do this two ways. You can use eq. 1.35 again with \( m_p = 938.3 \text{ MeV/c}^2 \), or you can just convert a-c using

\[ 1.00 \text{ MeV} = \frac{1}{c} \left( 1.602 \times 10^{-19} \text{ J} \right) = 5.34 \times 10^{-22} \text{ kg m/s} \]
#26.) continued

Either way, you get

- a.) $p = 9.38 \text{ MeV}/c$
- b.) $p = 540 \text{ MeV}/c$
- c.) $p = 1930 \text{ MeV}/c$

#28.) Consider the relativistic form of Newton's second law. Show that when $\vec{F}$ is parallel to $\vec{v}$,

$$ F = m \left(1 - \frac{v^2}{c^2}\right)^{-3/2} \frac{dv}{dt} $$

where $m$ is the mass of an object and $v$ is its speed.

The relativistic form of Newton's second law is (eq. 1.36a)

$$ \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt} \left[ \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \right] $$

Using the chain rule, (Assuming $\vec{F} || \vec{v}$)

$$ \frac{d}{dt} = \frac{dv}{dt} \frac{d}{dv} $$

$$ \therefore F = \frac{d}{dv} \left[ \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \frac{dv}{dt} $$

$$ = \left[ \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{m}{(1 - \frac{v^2}{c^2})^{3/2}} \left( - \frac{1}{2} \right) (- \frac{2v}{c^2}) \right] \frac{dv}{dt} $$
\[ \begin{align*}
\text{#28)} & \text{ continued} \\
& = \frac{mc^2(1-\frac{v^2}{c^2}) + uv^2}{c^2 \left( 1 - \frac{v^2}{c^2} \right)^{3/2}} \frac{dv}{dt} \\
\therefore \quad F &= m \left( 1 - \frac{v^2}{c^2} \right)^{-3/2} \frac{dv}{dt} \\
\vec{F} &= m \vec{a} \\
\end{align*} \]

#29.) A charged particle moves along a straight line in a uniform electric field \( \vec{E} \) with speed \( v \). If the motion and the electric field are both in the \( x \) direction,

a.) Show that the magnitude of the acceleration of the charge \( q \) is given by

\[ a = \frac{dv}{dt} = \frac{qE}{m} \left( 1 - \frac{v^2}{c^2} \right)^{3/2} \]

Just use the result of the last problem

\[ F = m \left( 1 - \frac{v^2}{c^2} \right)^{-3/2} \frac{dv}{dt} \]

And the Lorentz force equation

\[ \vec{F} = q \vec{E} + q \vec{v} \times \vec{B} = q \vec{E} \]

\[ \therefore m \left( 1 - \frac{v^2}{c^2} \right)^{-3/2} \frac{dv}{dt} = q \frac{E}{m} \]

\[ \Rightarrow \frac{dv}{dt} = \frac{qE}{m} \left( 1 - \frac{v^2}{c^2} \right)^{3/2} = a \]
b.) Discuss the significance of the dependence of the acceleration on the speed.

Another example that all velocities of massive objects must be less than c. If you graph a vs. v, you get

\[ a = \frac{\frac{dv}{dt}}{1 - \frac{v^2}{c^2}} \]

As v increases, it gets harder and harder to accelerate the object, and once v gets greater than c, the acceleration becomes imaginary, meaning that can't happen!

\[ \int_0^v \frac{dv'}{(1 - \frac{v'^2}{c^2})^{3/2}} = \frac{qE}{m} t \]

(1-b.) If the particle starts from rest at x=0, t=0, find the speed of the particle and its position after a time t has elapsed. Comment on the limiting value of v and x as \( t \to \infty \)

Start with

\[ a = \frac{qE}{m} \left(1 - \frac{v^2}{c^2}\right)^{3/2} \]

Separate variables

\[ \frac{dv}{(1 - \frac{v^2}{c^2})^{3/2}} = \frac{qE}{m} dt \]

Integrate

\[ \int_0^v \frac{dv'}{(1 - \frac{v'^2}{c^2})^{3/2}} = \frac{qE t}{m} \]
# 29. continued

Easiest thing to do is look that bad boy up in a table somewhere to get

\[ \frac{V}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{qE}{m} \]

\[ v^2 = \left( \frac{qE}{mc} \right)^2 \left( 1 - \frac{v^2}{c^2} \right) \]

\[ \Rightarrow \left[ 1 + \left( \frac{qE}{mc} \right)^2 \right] v^2 = \left( \frac{qE}{mc} \right)^2 \]

\[ \therefore v = \frac{qE}{m} \sqrt{1 + \left( \frac{qE}{mc} \right)^2} = \frac{qE}{mc} \sqrt{\frac{c}{\sqrt{mc^2 + qE^2}}} \]

as \( t \to \infty \), \( v \) asymptotically approaches the constant value

\[ v = c \quad \text{How about that?} \]

Now,

\[ v = \frac{dx}{dt} = \frac{qE}{mc} \sqrt{1 + \left( \frac{qE}{mc} \right)^2} \]

Again, separate variables and integrate

\[ \int_{x_0}^{x} dx = \frac{qE}{m} \int_{t_0}^{t} \frac{dt}{\sqrt{1 + \left( \frac{qE}{mc} \right)^2}} \]

Much easier.

let \( u = 1 + \left( \frac{qE}{mc} \right)^2 \quad \text{when} \ t \to \infty \), \( u = 1 \)

\[ t = \frac{1}{u} \quad \text{t} = t \quad \text{u} = 1 + \left( \frac{qE}{mc} \right)^2 \]

\[ \frac{du}{dt} = 2\frac{qE}{mc^2} = \left( \frac{mc^2}{qE} \right) \frac{d\frac{1}{u}}{dt} = cdt \]

\[ \int \frac{1}{c} = \int dt = t dt \]
\[ x = \frac{qE}{2m} \left( \frac{mc}{qE} \right)^2 \int_1^{\frac{qE}{mc}} du \left[ \frac{me^2}{1 - \frac{v^2}{c^2}} \right] \]

\[ x = \frac{c}{qE} \left[ \sqrt{\left( mc \right)^2 + \left( qEt \right)^2} - mc \right] \]

As \( t \to \infty \)

\[ x \to \frac{c}{qE} \left[ \sqrt{q^2 E^2 t^2} \right] = ct \]

As expected, for something moving with speed \( c \)!

#30.1) Recall that the magnetic force on a charge \( q \) moving with velocity \( \mathbf{v} \) in a magnetic field \( \mathbf{B} \) is equal to \( q \mathbf{v} \times \mathbf{B} \). If a charged particle moves in a circular orbit with a fixed speed \( v \) in the presence of a constant magnetic field, use Newton's second law to show that the frequency of its orbital motion is

\[ f = \frac{qB}{2\pi m} \left( 1 - \frac{v^2}{c^2} \right)^{1/2} \]

Set up the relativistic version of Newton's second law

eq. 1.36

\[ \mathbf{F} = q \mathbf{v} \times \mathbf{B} = \frac{d}{dt} \left[ \frac{m\mathbf{v}^2}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \]

We know that since the particle is moving in a circular orbit, \( \mathbf{v} \parallel \mathbf{B} \)

\[ \therefore q \mathbf{v} \mathbf{B} \hat{r} = \frac{d}{dt} \left[ \frac{m\mathbf{v}^2}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \quad \hat{r} \text{ is a unit vector pointing in radial direction} \]
The magnitude of \( \vec{v} \) remains constant, but not direction!

\[
\therefore \frac{d}{dt} \left[ \frac{m \vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} \right] = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{d\vec{v}}{dt}
\]

\( \frac{d\vec{v}}{dt} \) is just the centripetal acceleration

\[
\Rightarrow \frac{d\vec{v}}{dt} = \frac{v^2}{r} \hat{r}
\]

\[
\therefore qvB \hat{r} = \frac{m v^2}{r \sqrt{1 - \frac{v^2}{c^2}}} \hat{r}
\]

\[
\Rightarrow v = \frac{qB \sqrt{1 - \frac{v^2}{c^2}}}{m}
\]

The period is the time to complete one revolution

\[
T = \frac{2\pi r}{v} = \frac{2\pi r}{\frac{qB \sqrt{1 - \frac{v^2}{c^2}}}{m}} = \frac{2\pi m}{qB \sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
\therefore f = \frac{1}{T} = \frac{qB}{2\pi m} \left(1 - \frac{v^2}{c^2}\right)^{1/2}
\]

#31.) Show that the momentum of a particle having charge e moving in a circle of radius R in a magnetic field B is given by

\[ p = 300 BR \]

Where \( p \) is in MeV/c, \( B \) is in Teslas, and \( R \) is in meters

This problem is worked out for you in example 1.11, page 34.
Use eq. 1.38
\[ p = q \cdot BR \]
\[ = e \cdot BR \cdot 1.60 \times 10^{-19} \text{ BR } \frac{\text{kg m}}{s} \]
\[ \frac{1 \text{ MeV}}{c} = 5.34 \times 10^{-6} \frac{\text{kg m}}{s} \]
\[ \therefore p = 300 \text{ BR } \frac{\text{MeV}}{c} \]