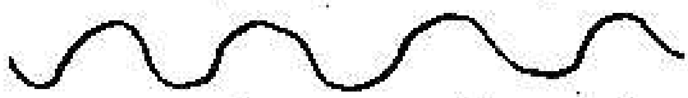
Waves: The 2C Experience

This handout covers the physics of waves and specifically Electromagnetic (such as light) waves, for those of you who didn't take 2C.

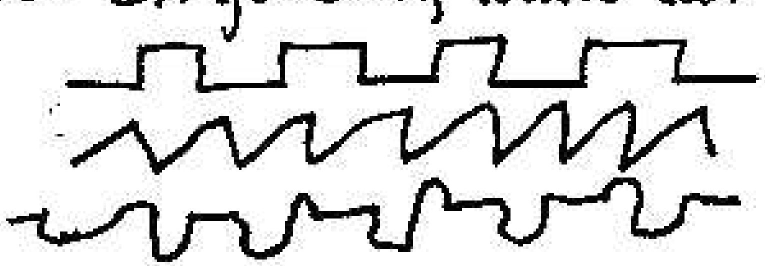
So what's a wave?

water waves, sound waves, shock waves – you hear about them every day. In general a wave is some kind of periodic pattern, a pattern that repeats itself. It can be a moving pattern, like the waves in the ocean, or a stationary pattern, like the waves of a vibrating guitar string.

Most waves look "wavy":

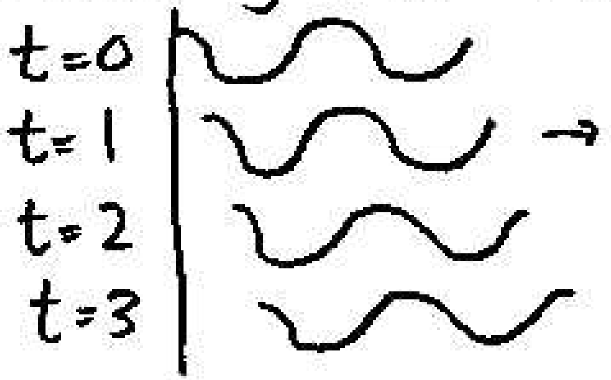


in Physics terms, they look like the graph of sine or cosine. In general, waves can be any shape:



but in physics <u>sine waves</u> are most common.

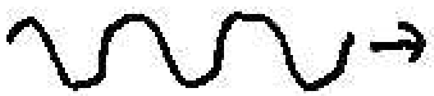
A traveling wave moves:



over time, the wave moves in some direction without changing shape (like ocean waves).

Traveling waves can be either transverse or longitudinal.

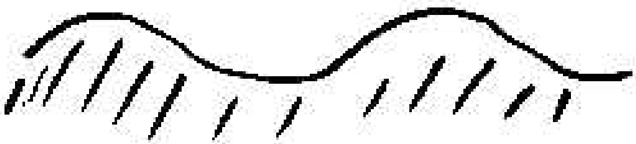
Transverse waves are "wavy" in a direction perpendicular to the direction of travel:



This wave moves right, but the undulations of the wave are oriented up and down.

This type includes water waves, waves on strings, and Electromagnetic (EM) waves.

Think about a water wave:

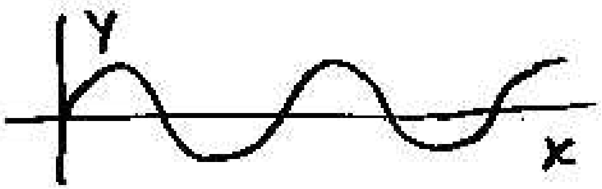


the wave moves along the water's surface, but the water particles themselves move up and down (and so do you when you sit in the water).

This is true of all transverse waves: the particles of the substance doing the "waving" move ⊥ to the direction of the wave.

There are also Longitudinal waves, like sound waves, in which the particles move along (parallel to) the direction the wave travels in.

A sine wave is a transverse wave:



it has the functional form

$$y(x,t) = A\sin(kx + \omega t + \phi)$$

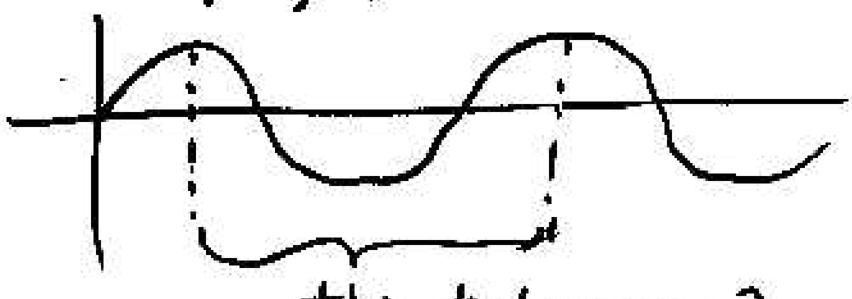
- x is x-coordinate
- A is the <u>amplitude</u>
- t is time
- k is called the <u>wave number</u>
- ω is called the <u>angular frequency</u>
- ϕ is called the phase

This wave travels along the x-axis, and any x-position in space will move up and down in y-direction sinusoidally.

k is related to the <u>wavelength</u>:

wavelength has the symbol λ, and is the distance between repetitions of the waves shape –

for example, the distance between peaks:



this distance is λ

by definition, $k = 2\pi/\lambda$.

ω is related to the <u>Period</u>:

The period, symbol T, is the time it takes for one wavelength to pass a given x coordinate.

$$\omega = \frac{2\pi}{T}.$$

ω is also related to the <u>frequency</u>, f:

f is the number of wavelengths that pass per unit time.

$$\omega = 2\pi f.$$

So, $f = \frac{1}{T}$.

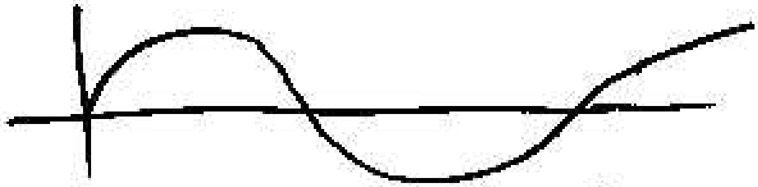
The <u>velocity</u> of the <u>wave</u> (how fast it travels along the x-direction) is <u>V</u>,

and it's given by $V = \lambda f = \frac{\omega}{k} = \frac{\lambda}{T}$

** lot's of interrelated formulas! **

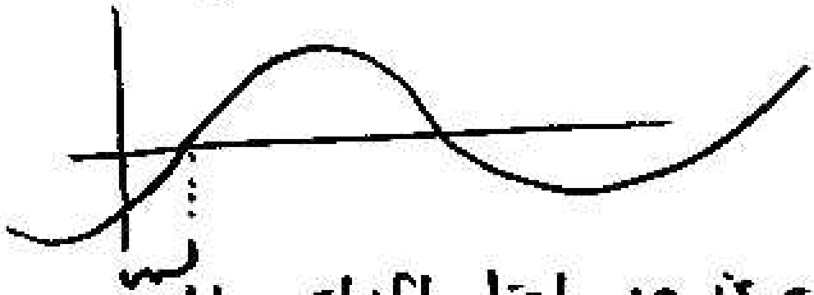
The phase ϕ is related to the shift of the wave along the x-axis relative to other waves.

A sine wave with $\phi = 0$ looks like:



at $t=0$

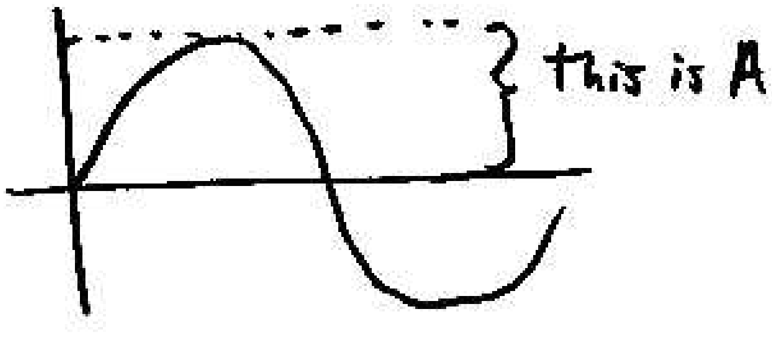
a sine wave with $\phi > 0$ will be shifted right at $t=0$:



the shift distance is given by $\frac{\lambda}{2\pi} \cdot \phi$

two waves that differ only in phase will always be shifted relative to each other by this amount, at all times.

Amplitude A gives the height of the peak measured from $y=0$:



Why? maximum of wave is at $\sin(kx + \omega t + \phi) = 1$, and here $y(x,t) = A$

$y(x,t)$ gives the shape of the wave by giving the height at any x-value. Note that it also depends on time.

A wave of the form $\sin(kx - \omega t)$

moves along the <u>positive</u> x-direction,

And a wave of the form $\sin(kx + \omega t)$

moves along the <u>negative</u> x-direction.

$\frac{dy}{dt}$ gives the <u>transverse</u> speed, i.e. the speed of the particles <u>in</u> the wave (like you bobbing up and down in the surf).

When two or more waves meet (say, 3 water waves, 2 waves on a string), they combine to form a new wave. It turns out that they combine by <u>adding</u>: if waves y_1, y_2, and y_3 meet,

$$y_{Total}(x,t) = y_1(x,t) + y_2(x,t) + y_3(x,t).$$

This is called the <u>Principle of Superposition</u>!

(by the way, they didn't <u>have</u> to add - i.e. it's not obvious from what I've said so far that they do.)

There are several special cases of 2 waves adding, where if they are sinusoidal waves, the result can be put in a simple mathematical form.

1st case: if $y_1 = A\sin(kx - \omega t + \phi)$

and $y_2 = A\sin(kx - \omega t)$

(i.e. they only differ by a phase)

Then you can prove that

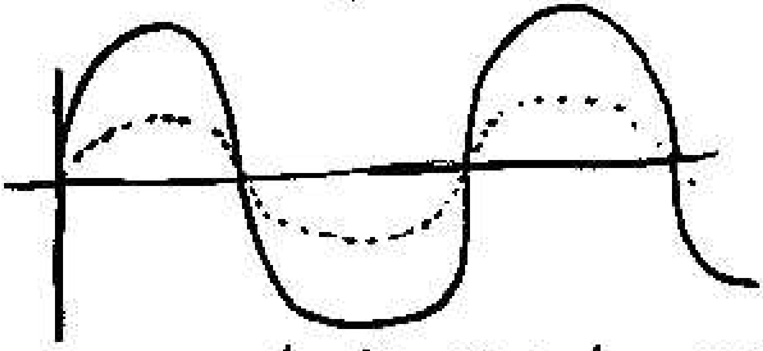
$$y_1 + y_2 = 2A\left(\cos\tfrac{1}{2}\phi\right)\left(\sin(kx - \omega t + \tfrac{1}{2}\phi)\right)$$

which is also a sinusoidal wave - 2 sine waves in this case add to make one sine wave!

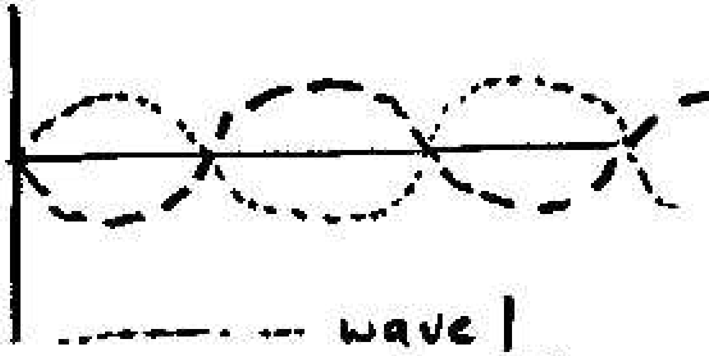
Now, the amplitude $= 2A\cos\frac{1}{2}\phi$, which depends on ϕ. So the height of the new wave depends on the phase difference of the two original waves. This has the special name interference. We say y_1 and y_2 are interfering. If $\phi = 0$, the waves are in step with each other, and the amplitude has it's maximum value of $2A$. If $\phi = \pi$, the amplitude is zero. If $\phi = 2\pi$, the amplitude is back up to $2A$ (ignore the $-$ sign). For values of ϕ in between 0 and π, and π and 2π, the amplitude is somewhere in between 0 and $2A$.

So, depending on how far the initial 2 waves are out of step, the resultant wave's amplitude can go from 0 to 2A, or from no wave at all to a wave twice the height of the original waves.

1. when $\phi = 0$, the waves are exactly in step, so they combine to make a wave twice their height.



2. when $\phi = \pi$, the waves are out of step by exactly $\frac{1}{2}\lambda$ (check this!), and they combine to form no wave.



3. when $\phi = 2\pi$, the waves are back in step and they form the 2A amplitude wave again.

In general, the waves are in step for ϕ = an even integer multiple of ϕ ($2\pi, 4\pi, 6\pi, 0\pi \ldots$) and the 2A wave is formed. This is called

Constructive Interference

When the two waves are out of step by $\frac{1}{2}\lambda$, in general ϕ = an <u>odd</u> multiple of π (3π, 5π, 7π, π ...) and the resultant wave has zero amplitude. This is called

Destructive Interference.

For some intermediate value of ϕ, you get some interference intermediate between these.

<u>Case 2</u>. Another case where 2 waves adding gives a simple result is when the waves are identical except for the direction of travel:

if $y_1 = A \sin(kx - \omega t)$ (moves in + x direction)

$y_2 = A \sin(kx + \omega t)$ (in −x dir.)

Then $y_1 + y_2 = 2A \sin kx \cos \omega t$.

This wave is called a <u>standing</u> <u>wave</u>.

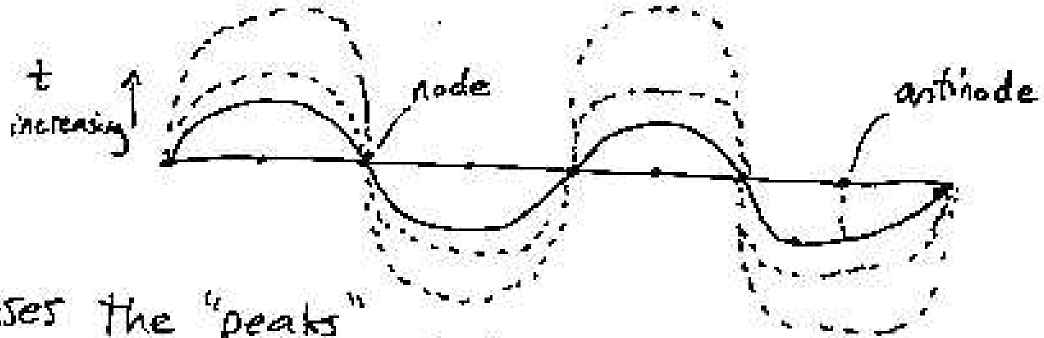
Why? It turns out that only waves of the form $y = f(kx \pm \omega t)$ (f is some function) are moving — all other waves do not move. The wave given above is a common example of a non-moving wave, so it has been given the name "standing wave". This is how waves on guitar or piano strings look.

New marker! If any of these pages are unreadable, let me know (soconnor@physics.ucsd.edu)

Let's look at standing waves more closely:

- Standing waves have <u>nodes</u> - locations where the amplitude is always zero. These locations are at $\sin kx = 0$, or $x = \frac{n\pi}{k}$, n integer.
- standing waves are completely flat at some times, where $\cos \omega t = 0$, and reach a maximum amplitude at other times, where $\cos \omega t = \pm 1$.

A graph of a standing wave looks like:



As time increases the "peaks" of the wave oscillate up and down.

Points for which the amplitude is always zero are called nodes. Points for which the wave has its maximum value are <u>antinodes</u> (the peaks of the wave).

Important! Note that for a standing wave, the endpoints stay where they are in space - in general, they are also restricted to be either a node or an antinode. Because of this, only a <u>half-integral number of wavelengths</u> can fit between the endpoints - like $5\frac{1}{2}\lambda$, $7\frac{1}{2}\lambda$, 6λ, 4λ

We're done with the general theory of waves. There are several topics covered in 2C that show examples of these waves that I won't cover here. If they come up in class, get help from Sean or Matt or read up on it in Haliday & Resnick. These topics include:

- Standing waves in strings (ex. = piano strings)
- Longitudinal waves - most common example is sound waves.
- Beats
- Doppler effect
- Standing sound waves in pipes

Why didn't I cover longitudinal waves? In general they are important, but for this class we're only going to talk about transverse waves. The most common transverse wave you see every day is light - Light is a form of transverse wave called an Electromagnetic Wave.

EM waves include radio waves, microwaves, all visible light, infrared, ultraviolet, and mucho more others (that you see and use every day). The waves differ in their frequency, and you can make an EM "spectrum" by showing which waves are in what frequency range:

Frequency (Hz):	10^7	10^8	10^{13}	10^{14}	10^{16}	10^{18}
	Radio waves	T.V. Channels	infra-red	visible light	ultra violet	x-rays

Visible light only takes up a tiny slice of the EM spectrum.

So what's an EM wave?

Remember a general transverse wave?

$y = A\sin(kx - \omega t + \phi)$.

For any point x along the x-axis, this equation gave the height y as a function of time - the height oscillated up and down. What if instead of height, I told you that "y" meant something else? Say, y means temperature. Then at any point on the x axis, this equation gives you the temperature as a function of time - the temperature oscillates up and down. So Instead of a wave with a height, this wave has a "temperature height", making a "temperature wave." A wave could be made of any quantity, like temperature, pressure (which is what sound waves are made of), and in the case of EM waves, E and B fields.

For an EM wave, there are actually two functions that look like $y = A\sin(kx - \omega t + \phi)$. For each point x, one gives the value of the electric field, another gives the value of the magnetic field:

They look like:

$$E = E_m \sin(kx - \omega t)$$
$$B = B_m \sin(kx - \omega t)$$

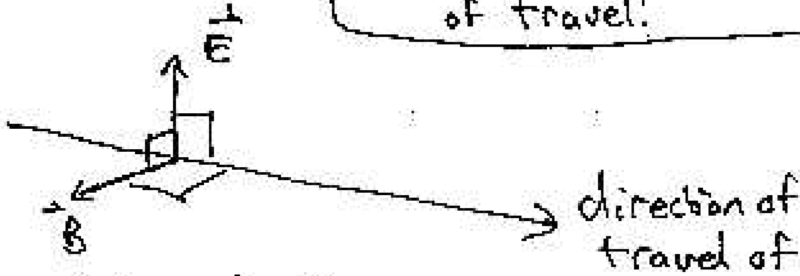
B_m and E_m are the amplitudes

So the two quantities E and B vary sinusoidally in time for a given x, and they form an "electromagnetic" wave that moves in space just like the temperature waves.

There is one difference between E and B and Temperature, however. E and B are vectors. So the $\vec{E}$ and $\vec{B}$ fields have a magnitude (given by the sine functions) and a direction each. What are the directions?

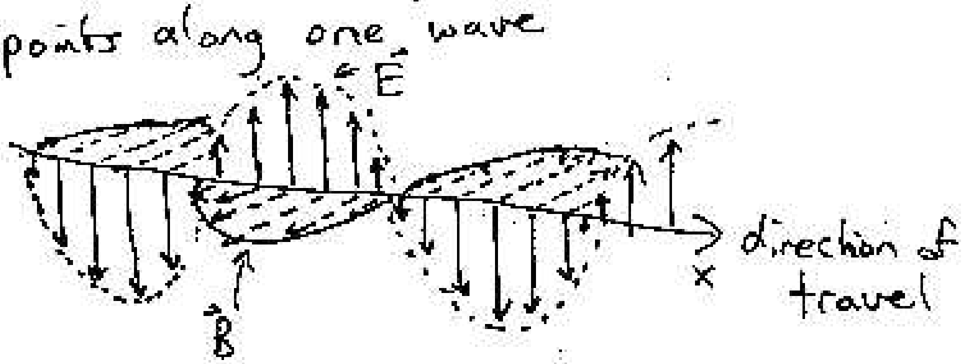
$\boxed{\text{note: } \vec{E} \times \vec{B} = \text{direction of travel!}}$

Both are perpendicular to each other and to the direction of travel.



So since $\vec{E}$ and $\vec{B}$ vary transversely to the direction of travel, they are called transverse waves, and the two together make up an EM wave!

A drawing of many points along one wave looks like:



Each varies sinusoidally in space, and the entire wave moves forward along the x-axis.

It turns out that all EM waves have the same speed, which is called the speed of light, with symbol $\underline{c}$.

$$c = 3 \times 10^8 \frac{m}{s} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

And it also so happens that $\frac{E_m}{B_m} = c$.

So different EM waves are called different because they have different frequency, and because they all have speed c, their wavelengths are also different according to $\lambda f = c$.

Let's write out in vector form what an EM wave looks like:

A wave of the form

$$\vec{E} = E_m \sin(kx - \omega t + \phi)\,\hat{y}$$
$$\vec{B} = B_m \sin(kx - \omega t + \phi)\,\hat{z}$$

is a wave traveling in the $+\hat{x}$ direction.

Note that the direction of $\vec{E} \times \vec{B}$ is in $+\hat{x}$ direction.

Some details about EM waves that I won't cover here can also be found in Haliday and Resnicke. These shouldn't be vital to learning 2D, but if you have questions, ask one of us. These include:

- The Poynting vector $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$.
- Radiation pressure
- Polarization - direction of polarization of an EM wave is defined to be the direction of the $\vec{E}$ component.
- Reflection, refraction, lenses - There's a lot of detailed information on these topics done in 2C, and some basic facts may be useful. To avoid doing a lot of unnecessary details, instead of covering this now I'd prefer to explain needed concepts as they come up. For information on these, check out H&R Chapters 34 & 35.

2 details I will cover are

Interference and Diffraction

$\downarrow$ See Ch. 36 in H&R $\qquad$ $\downarrow$ See Ch. 37

Interference -

You've already learned how 2 sine waves can <u>interfere</u>, and if you aren't drooling under the weight of all this new knowledge yet, I'll mention it again:

- 2 waves <u>constructively</u> <u>interfere</u> if they are <u>in</u> <u>step</u> or out of step by an <u>even</u> multiple of ~~wavelengths~~. π (of phase
- 2 waves <u>destructively</u> <u>interfere</u> if they are out of step by an <u>odd</u> multiple of π.

The first is equivalent to being shifted relatively by one or more wavelengths, and the second to an odd number of half-wavelengths.

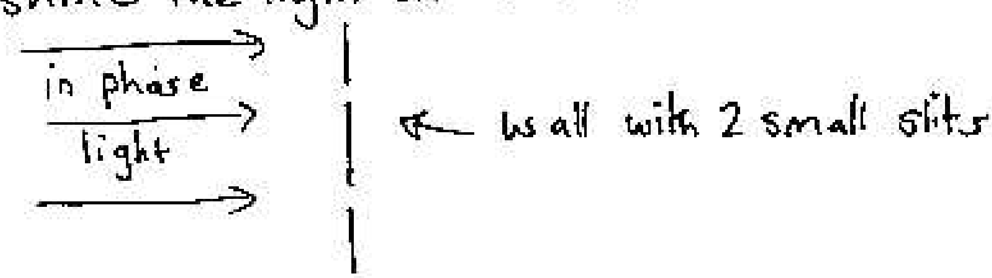
Interference comes from shifts in phase, and there are 3 common ways for 2 light rays that are the same phase to become out of phase with each other.

<u>The first way</u>:

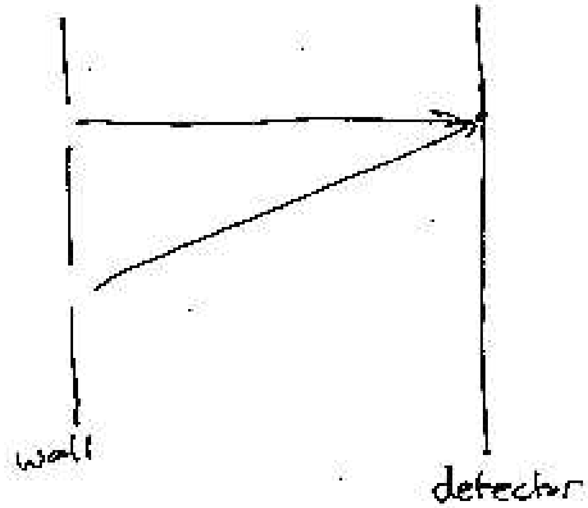
Say two light rays are in phase at one point in space. If the two light rays travel <u>different</u> <u>paths</u> to get to some other point, they will be <u>out</u> of phase at that point if the paths were of different lengths. If the path length difference is L, their phase difference will be $\frac{L}{\lambda}(2\pi)$.

The most famous example of this is the "2-slit" experiment. Imagine a beam of light, made of a bunch of EM waves, emitted from a source with all waves in phase. We shine the light on a wall that has 2 slits cut into it:

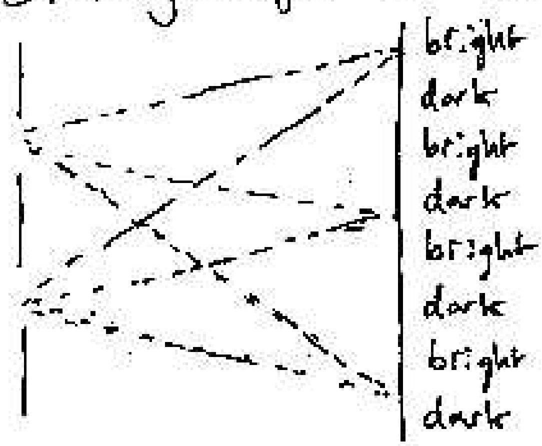
(top view)



The light that goes through each slit is in phase, so the equations for the EM waves going through each slit are identical. Then we put some kind of detector, like photographic film, on the far side of the wall that can record the amplitude of the combined wave it receives. The combined wave is the sum of waves coming from each slit.

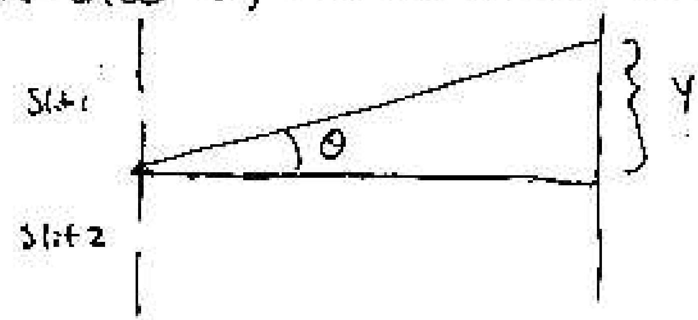


At some arbitrary point on the detector, then, the waves from both slits interfere to create a combined wave seen on the screen. If they constructively interfere, the resultant wave has the highest possible amplitude, and a bright spot is recorded on the detector. If they destructively interfere, the resultant wave has no amplitude, and a dark spot is seen. The result for all points on the detector is a series of alternating bright and dark spots, which we call "fringes":



Why? Because the path length difference between the light rays alternates between being a multiple of the wavelength and "off" by ½ of a wavelength.

We can get an equation for the locations of the bright fringes, which are called maxima, and for dark fringes, or minima. If the two slits are very thin, are separated by a distance d, and we make this angle θ: (17)



Then maxima will appear at the angle given by

$$d \sin\theta = m\lambda \quad , \quad m \text{ integer} = 0, 1, 2, 3 \ldots$$

And minima will appear at

$$d \sin\theta = \left(m + \tfrac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \ldots$$

I'll spare you the details of the derivation (see p. 906-908 of H&R for details), but I will tell you the main idea. Two identical waves interfere because they travel different path lengths. If the path length difference is an <u>integer multiple</u> of the wavelength, they interfere <u>constructively</u>. If the length difference is "off" by $\frac{1}{2}\lambda$ (it's an o<u>dd</u> multiple of a half-wavelength), they interfere <u>destructively</u>.

This, for our purposes, is the most important method to get two identical waves to interfere. But there are two others.

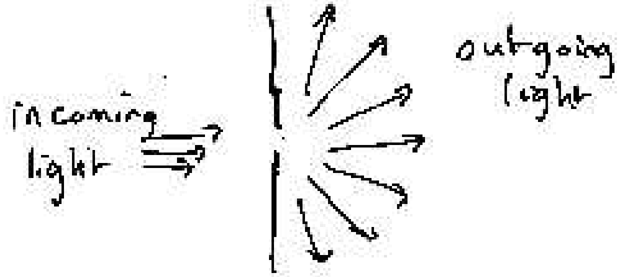
2nd way: If light is not traveling through a vacuum but instead through some transparent chunk o' matter, the atoms in the matter interfere with the light and reduce its wavelength. Each substance reduces the wavelength by some characteristic ~~amount~~ amount, given by a constant called the "index of refraction" for that material. The constant has the symbol $\underline{n}$, and for light of wavelength λ, the wavelength is reduced to $\frac{\lambda}{n}$ in the material.

So, if two identical waves go through different chunks of matter, they will change wavelength. When they exit the chunks of matter, they will then have a phase difference. (see H&R p. 904 for details).

3rd way: If a ray of light traveling in a chunk of matter bounces off another chunk of matter with a higher index of refraction, it will pick up a phase shift of π. (see H&R p. 912 for details).

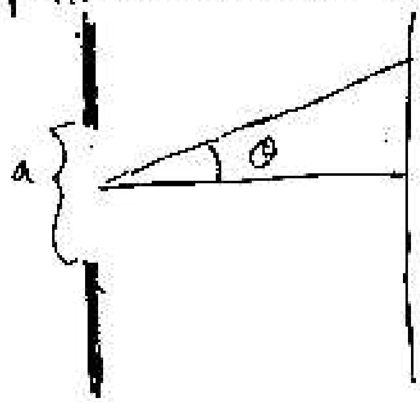
We're now done with interference, so let's boogie on over to diffraction!

Diffraction: In the 2-slit set-up I explained for the interference effect, I implied that the light coming out of the slit travels in all directions:



The fact that it does this is called diffraction. In general, light traveling in one direction is bent, or spread out, by a small hole or even a sharp edge. The smaller the hole, the more pronounced this effect is. This is diffraction.

For one slit, light spreads out due to diffraction. Because of this, light going through different parts of the slit can interfere with each other to create an interference pattern similar to the one described for two slits.



If the slit has width a, there will be interference minima at the angles given by $a \sin \theta = m\lambda$,

$$m = 1, 2, 3, 4, \ldots$$

This is called a "diffraction pattern". It has the same physical origin as the two slit interference pattern, namely interference between two rays of light because of path length differences, but in this case the pattern is from the cumulative effect of all the light rays going through one slit interfering with each other.

In general when light goes through several slits, both an interference pattern and a diffraction pattern will be seen, superimposed. For details on this pattern, see H&R 37 sections 1 & 2.

Two examples of diffraction are listed in H&R Ch. 37, because they are seen or used in physics research:

- diffraction from a circular aperture (hole): this is a source of error in telescopes when stargazing.
- "diffraction gratings": these can be used to determine the wavelength of light if it is unknown.

For details, check Ch. 37 in H&R. I won't go into detail on them here.

And that's all there is to it!

This handout isn't meant to provide everything you need to know from 2C - it's only meant as a starting point. Don't get discouraged if it didn't make much sense in parts - go to the book or to the T.A.'s for help. I condensed 5 chapters here into 20 pages, so I expect you to come talk to one of us or go to the book to get the whole story.

Good luck,

- Sean